C Appendix: Trading in Segmented Markets

C.1 General set-up

Our framework can provide insights about trade in segmented markets as well. Markets are segmented when investors, such as hedge funds and asset management firms, trade in some markets but not in others. Although segmented, markets can be connected, in the sense agents are able to trade in multiple venues at the same time. To study the implications in segmented markets, we extend our model in the following way.

We consider an economy in which there are N trading posts connected in a network g. At each trading post, I, there exist n^I risk-neutral dealers. The entire set of dealers is denoted $\mathcal{N} = \bigcup_{I=N}^{N} I$. Each dealer $i \in I$ can trade with other dealers in his own trading post and with dealers at any trading post J that is connected with the trading post I by a link IJ. Essentially, the link IJ represents a market in which dealers at trading posts I and J can trade with each other. However, they have access to trade in other markets at the same time. Let g^I denote the set of trading posts that are linked with I in the network g, and $m^I \equiv \left|g^I\right|$ represent the number of I's links.

As before, dealers trade a risky asset in zero net supply, and all trades take place at the same time. Each dealer is uncertain about the value of the asset. In particular, a dealer's value for the asset is given by θ^i, which is a random variable normally distributed with mean 0 and variance σ_θ^2. Moreover, we consider that values are interdependent across all dealers. In particular, $\mathcal{V}(\theta^i, \theta^j) = \rho\sigma_\theta^2$ for any two agents $i, j \in \mathcal{N}$. Each dealer receives a private signal, $s^i = \theta^i + \varepsilon^i$, where $\varepsilon^i \sim IID\, N(0, \sigma_\varepsilon^2)$ and $\mathcal{V}(\theta^j, \varepsilon^i) = 0$, for all i and j.

A dealer $i \in I$ seeks to maximize her final wealth

$$\sum_{J \in g^I} q_{IJ}^i \left(\theta^i - p_{IJ}\right),$$

where q_{IJ}^i is the quantity traded by dealer i in market IJ, at a price p_{IJ}. Similarly to the OTC model, the trading strategy of the dealer i with signal s^i is a generalized demand function $\mathbf{Q}^i : R^{m^i} \to R^{m^i}$ which maps the vector of prices, $\mathbf{p}_{g^I} = (p_{IJ})_{J \in g^I}$, that prevail in the markets in which dealer i participates in network g into a vector of quantities she wishes to trade

$$\mathbf{Q}^i(s^i; \mathbf{p}_{g^I}) = \left(Q_{IJ}^i(s^i; \mathbf{p}_{g^I})\right)_{J \in g^I},$$

where $Q_{IJ}^i(s^i; \mathbf{p}_{g^I})$ represents her demand function in market IJ.

Apart from trading with each other, dealers also serve a price-sensitive customer base. In particular, we assume that for each market IJ, the customer base generates a downward sloping demand

$$D_{IJ}(p_{IJ}) = \beta_{IJ} p_{IJ}, \tag{C.1}$$

with an arbitrary constant $\beta_{IJ} < 0$. The exogenous demand (C.1) ensures the existence of the equilibrium when agents are risk neutral, and facilitates comparisons with the OTC model.

The expected payoff for dealer $i \in I$ corresponding to the strategy profile $\left\{\mathbf{Q}^i\left(s^i; \mathbf{p}_{g^I}\right)\right\}_{i \in \mathcal{N}}$ is

$$E\left[\sum_{J \in g^I} Q_{IJ}^i(s^i; \mathbf{p}_{g^I}) \left(\theta^i - p_{IJ}\right) | s^i\right]$$

where p_{IJ} are the prices for which all markets clear. That is, prices satisfy

$$\sum_{i \in I} Q^i_{IJ}\left(s^i; \mathbf{p}_{g^I}\right) + \sum_{j \in J} Q^j_{IJ}\left(s^j; \mathbf{p}_{g^J}\right) + \beta_{IJ} p_{IJ} = 0, \forall\ IJ \in g. \tag{C.2}$$

C.2 Equilibrium concept

As in the OTC game, we use the concept of Bayesian Nash equilibrium. For completeness, we reproduce it below.

Definition 3 *A Linear Bayesian Nash equilibrium of the segmented market game is a vector of linear generalized demand functions* $\left\{\mathbf{Q}^i\left(s^i; \mathbf{p}_{g^I}\right)\right\}_{i \in \mathcal{N}}$ *such that* $\mathbf{Q}^i(s^i; \mathbf{p}_{g^I})$ *solves the problem*

$$\max_{(Q^i_{IJ})_{J \in g^I}} E\left\{\left[\sum_{J \in g^I} Q^i_{IJ}(s^i; \mathbf{p}_{g^I})\left(\theta^i - p_{IJ}\right)\right] |s^i\right\}, \tag{C.3}$$

for each dealer i*, where the prices* p_{IJ} *satisfy (C.2) .*

A dealer i chooses a demand function in each market IJ, in order to maximize her expected profits, given her information, s^i, and given the demand functions chosen by the other dealers.

C.3 The Equilibrium

In this section, we outline the steps for deriving the equilibrium in the segmented market game for any network structure. First, we derive the equilibrium strategies as a function of posterior beliefs. Second, we construct posterior beliefs that are consistent with dealers' optimal choices. In the OTC game we used the conditional guessing game as an intermediate step in constructing beliefs. Here, we employ the same line of reasoning, although we do not explicitly introduce the conditional guessing game structure that would correspond to the segmented market game.

C.3.1 Derivation of demand functions

We conjecture an equilibrium in demand functions, where the demand function of dealer i in market IJ is given by

$$Q^i_{IJ}(s^i; \mathbf{p}_{g^I}) = t^I_{IJ}(y^I_{IJ} s^i + \sum_{K \in g^I} z^I_{IJ,IK} p_{IK} - p_{IJ}) \tag{C.4}$$

for any $i \in I$ and J. As evident in the notation, we consider that all dealers at trading post I are symmetric in their trading strategy, and weigh in same way the signal they receive and the prices they trade at. Nevertheless, they end up trading different quantities, as they have different realizations of the signal.

We solve the optimization problem (C.3) pointwise. That is, for each realization of the vector of signals, $\mathbf{s}$, we solve for the optimal quantity q^i_{IJ} that each dealer $i \in I$ demands in market IJ. Given the conjecture (C.4) and the market clearing conditions (C.2), the residual inverse demand function of dealer i in market IJ is

$$p_{IJ} = -\frac{t_{IJ}^I y_{IJ}^I \sum\limits_{k\in I, k\neq i} s^k + t_{IJ}^J y_{IJ}^J \sum\limits_{k\in J} s_k + (N_I - 1) \sum\limits_{L\in g^I, L\neq J} t_{IJ}^I z_{IJ,IL}^I p_{IL} + N_J \sum\limits_{L\in g^J, L\neq I} t_{IJ}^J z_{IJ,JL}^J p_{JL} + q_{IJ}^i}{(N_I - 1)\, t_{IJ}^I \left(z_{IJ,IJ}^I - 1\right) + N_J t_{IJ}^J \left(z_{IJ,IJ}^J - 1\right) + \beta_{IJ}}. \tag{C.5}$$

Denote

$$I_i^J \equiv -\frac{t_{IJ}^I y_{IJ}^I \sum\limits_{k\in I, k\neq i} s^k + t_{IJ}^J y_{IJ}^J \sum\limits_{k\in J} s_k + (N_I - 1) \sum\limits_{L\in g^I, L\neq J} t_{IJ}^I z_{IJ,IL}^I p_{IL} + N_J \sum\limits_{L\in g^J, L\neq I} t_{IJ}^J z_{IJ,JL}^J p_{JL}}{(N_I - 1)\, t_{IJ}^I \left(z_{IJ,IJ}^I - 1\right) + N_J t_{IJ}^J \left(z_{IJ,IJ}^J - 1\right) + \beta_{IJ}} \tag{C.6}$$

and rewrite (C.5) as

$$p_{IJ} = I_i^J - \frac{1}{(N_I - 1)\, t_{IJ}^I \left(z_{IJ,IJ}^I - 1\right) + N_J t_{IJ}^J \left(z_{IJ,IJ}^J - 1\right) + \beta_{IJ}} q_{IJ}^i. \tag{C.7}$$

The uncertainty that dealer i faces about the signals of others is reflected in the random intercept of the residual inverse demand, I_i^J, while her capacity to affect the price is reflected in the slope $-1/\left((N_I - 1)\, t_{IJ}^I \left(z_{IJ,IJ}^I - 1\right) + N_J t_{IJ}^J \left(z_{IJ,IJ}^J - 1\right) + \beta_{IJ}\right)$. In the segmented markets game, however, the random intercept I_i^J reflects not only the signals of the dealers at the trading post J, but also the signals of the other dealers at the trading post I.

Then, solving the optimization problem (C.3) is equivalent to finding the vector of quantities $\mathbf{q}^i = \mathbf{Q}^i(s^i; \mathbf{p}_{g^I})$ that solve

$$\max_{(q_{IJ}^i)_{j\in g^I}} \sum_{J\in g^I} q_{IJ}^i \left(E\left(\theta^i | s^i, \mathbf{p}_{g^I}\right) - \left(I_i^J - \frac{q_{IJ}^i}{(N_I - 1)\, t_{IJ}^I \left(z_{IJ,IJ}^I - 1\right) + N_J t_{IJ}^J \left(z_{IJ,IJ}^J - 1\right) + \beta_{IJ}} \right) \right)$$

From the first order conditions we derive the quantities q_{IJ}^i that dealer $i \in I$ trades in each market IJ, for each realization of $\mathbf{s}$, as

$$2\frac{1}{(N_I - 1)\, t_{IJ}^I \left(z_{IJ,IJ}^I - 1\right) + N_J t_{IJ}^J \left(z_{IJ,IJ}^J - 1\right) + \beta_{IJ}} q_{IJ}^i = I_i^J - E\left(\theta^i | s^i, \mathbf{p}_{g^I}\right),$$

This implies that the optimal demand function

$$Q_{IJ}^i(s^i; \mathbf{p}_{g^i}) = -\left((N_I - 1)\, t_{IJ}^I \left(z_{IJ,IJ}^I - 1\right) + N_J t_{IJ}^J \left(z_{IJ,IJ}^J - 1\right) + \beta_{IJ}\right) \left(E(\theta^i \left| s^i, \mathbf{p}_{g^I}\right.) - p_{IJ}\right) \tag{C.8}$$

for each dealer i in market IJ.

Further, given our conjecture (C.4), equating coefficients in equation (C.8) implies that

$$E(\theta^i \left| s^i, \mathbf{p}_{g^I}\right.) = y_{IJ}^I s^i + \sum_{K\in g^I} z_{IJ,IK}^I p_{IK}.$$

However, the projection theorem implies that the belief of each dealer i can be described as a unique linear combination of her signal and the prices she observes. Thus, it must be that $y^I_{IJ} = y^I$, and $z^I_{IJ,JK} = z^I_{IK}$ for all I, J, and K. In other words, the posterior belief of a dealer i is given by

$$E(\theta^i \left| s^i, \mathbf{p}_{g^I} \right.) = y^I s^i + \mathbf{z}_{g^I} \mathbf{p}_{g^I}, \tag{C.9}$$

where $\mathbf{z}_{g^I} = \left(z^I_{IJ}\right)_{J \in g^I}$ is a *row* vector of size m^i. Then, we obtain that the trading intensity of dealer at trading post I satisfies

$$t^I_{IJ} = (N_I - 1)\, t^I_{IJ} \left(1 - z^I_{IJ}\right) + N_J t^J_{IJ} \left(1 - z^J_{IJ}\right) - \beta_{IJ}. \tag{C.10}$$

If we further substitute this into the market clearing conditions (C.2) we obtain the price in market IJ as follows

$$p_{IJ} = \frac{t^I_{IJ} \left(\sum_{i \in I} E\left(\theta^i | s^i, \mathbf{p}_{g^I}\right)\right) + t^J_{IJ} \left(\sum_{j \in J} E\left(\theta^j | s^j, \mathbf{p}_{g^J}\right)\right)}{N_I t^I_{IJ} + N_J t^J_{IJ} - \beta_{IJ}}. \tag{C.11}$$

From (C.10) and the analogous equation for t^J_{IJ}, it is straightforward to derive the trading intensity that dealers at trading post I and J have. This implies that we can obtain the price in each market IJ as

$$p_{IJ} = w^I_{IJ} \left(\sum_{i \in I} E\left(\theta^i | s^i, \mathbf{p}_{g^I}\right)\right) + w^J_{IJ} \left(\sum_{j \in J} E\left(\theta^j | s^j, \mathbf{p}_{g^J}\right)\right), \tag{C.12}$$

where

$$w^I_{IJ} \equiv \frac{z^J_{IJ} - 2}{(N_J + N_I - 1)\, z^I_{IJ} z^J_{IJ} - 2\,(N_I - 1)\, z^I_{IJ} - 2\,(N_J - 1)\, z^J_{IJ} - 4}.$$

This expression is useful to relate the belief of a dealer $i \in I$ to the beliefs of other dealers at the same trading post, and at trading posts that are connected to I.

C.3.2 Derivation of beliefs

We follow the same solution method that we developed in Section 3.1. As before, the key idea is to reduce the dimensionality of the problem and use our conjecture about demand functions to derive a fixed point in beliefs, instead of the fixed point (C.8).

In the OTC game we constructed each dealer's equilibrium belief as a linear combination of the beliefs of her neighbors in the network. For this, we introduced the conditional guessing game. The conditional guessing game was a useful intermediate step in making the derivations more transparent, as well as an informative benchmark about the role of market power for the diffusion of information.

In the segmented market game it is less straightforward to formulate the corresponding conditional guessing game. Since there are multiple dealers at each trading post, it is not immediate how each dealer forms her guess. In particular, we would need to make additional assumptions about the linear combination of the guesses of dealers in the same trading post and dealers of the neighboring trading post, that each agent can condition her guess on.

Thus, in the segmented market game we construct beliefs directly as linear combinations of signals. We conjecture that for each dealer $i \in I$, her belief is an affine combination of the signals of all dealers in the economy

$$E\left(\theta^i | s^i, \mathbf{p}_{g^I}\right) = \bar{v}_{II}^I s^i + \sum_{K=1}^{N} v_{IK}^I S^K, \tag{C.13}$$

where $S^K = \sum_{k \in K} s^k$, $\forall K$. This further implies that

$$\sum_{i \in I} E\left(\theta^i | s^i, \mathbf{p}_{g^I}\right) = \bar{v}_{II}^I S^I + N_I \sum_{K=1}^{N} v_{IK}^I S^K.$$

Before we derive the fixed point equation for beliefs, it is useful to write (C.12) in matrix form, for each trading post I. For this we introduce some more notation. Unless specified otherwise, in the notation below we keep I fixed and vary $J \in \{1, ..., N\}$. Let $\mathbf{p}^I$ be a $N-$column vector with elements p_{IJ} if $IJ \in g$, and 0 otherwise. Let $\mathbf{z}^I$ be a $N-$column vector with elements z_{IJ}^I if $IJ \in g$, and 0 otherwise. Similarly, let $\mathbf{w}^I$ be the $N-$column vector with elements w_{IJ}^I if $IJ \in g$, and 0 otherwise, while W_I be a matrix with elements w_{IJ}^J on diagonal if IJ have a link, and 0 otherwise (all elements off-diagonal are 0, as well). Further, let $\mathbf{v}^I$ be the $N-$row vector with elements v_{IJ}^I, and $\bar{\mathbf{v}}^I$ be the $N-$row vector with elements $\bar{v}_{II}^I$ at position I and 0 otherwise. Let V be the square matrix with rows $\mathbf{v}^I$, and $\bar{V}$ be the matrix with rows $\bar{\mathbf{v}}^I$. Let $\mathbf{S}$ be the $N-$column vector with elements S^I. Let N be a square matrix with elements n^I on diagonal and 0 otherwise. Let $\mathbf{1}$ be the $N-$column vector of ones.

Substituting our conjecture for beliefs (C.13) in the equation for the price (C.12), we obtain the vector of prices which dealers at each trading post I are trading as

$$\mathbf{p}^I = \mathbf{w}^I \left(\bar{\mathbf{v}}^I + n^I \mathbf{v}^I\right) \mathbf{S} + W^I \left(\bar{V} + \mathsf{N} V\right) \mathbf{S}.$$

We are now ready to formalize the result.

Proposition C.1 *There exists an equilibrium in the segmented markets game if the following system of equations*

$$\mathbf{v}^I = \left(\mathbf{z}^I\right)^\top \left(\mathbf{w}^I \left(\bar{\mathbf{v}}^I + n^I \mathbf{v}^I\right) + W^I \left(\bar{V} + \mathsf{N} V\right)\right) \mathbf{1}, \forall I \tag{C.14}$$

and

$$\bar{v}_{II}^I = y^I, \forall I$$

admits a solution in $\mathbf{v}^I$*, for each* I*.*

Proof. As for the OTC game, the proof is constructive. Note that showing that equation (C.14) admits a solution is equivalent to showing that there exists a fixed point in $\mathbf{v}^I$. This is because, the projection theorem implies that $\mathbf{z}^I$, and inherently, $\mathbf{w}^I$ are a function of $\mathbf{v}^I$.

Let $\mathbf{v}^I$ be a fixed point of (C.14) and $\bar{v}_{II}^I = y^I$, for each I. We construct an equilibrium for the segmented-market game with beliefs given by (C.13), as follows. We choose conveniently $\mathbf{z}^I$ and $\mathbf{w}^I$ such that

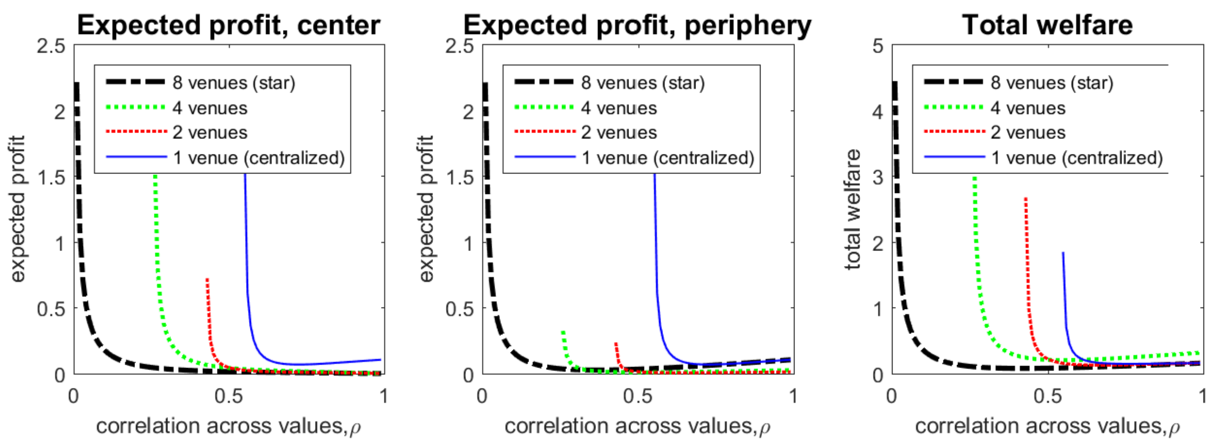
$$E\left(\theta^i | s^i, \mathbf{p}_{g^I}\right) = y^I s^i + \left(\mathbf{z}^I\right)^\top \left(\mathbf{w}^I \left(\bar{\mathbf{v}}^I + n^I \mathbf{v}^I\right) + W^I \left(\bar{V} + \mathsf{N} V\right)\right) \mathbf{S}$$

is satisfied. Then, it follows that the prices given by (C.11) and demand functions given by (C.8) is an equilibrium of the OTC game. ■

The derivation we have outlined above also highlights the main technical difficulty of the segmented market game relative to the OTC game. That is, the signals of dealers in the same trading post obscure the (sum of) beliefs of the dealers in neighboring trading posts, such that a dealer can no longer invert the prices she observes and infer what his neighbors posteriors.

C.4 Welfare and expected profit

To complement our example in the main text, we illustrate with the following figure how expected profit and welfare changes with market segmentation. We leave the detailed analysis for the reader and highlight only two interesting observations. First, as trading intensities were not monotonic for the periphery in the degree of segmentation, expected profit is not monotonic either. Also, total welfare is also not monotonic in segmentation.



D Appendix: The Price-Discovery Game

In this section, we guide the reader to better connect our OTC game to the real world features of trading in OTC markets. In the OTC game dealers' trading strategies are represented by generalized demand curves, and all trades take place simultaneously. This is a very tractable and rich theoretical structure, and we consider the one-shot OTC game as a reduced form representation of how equilibrium prices and quantities are determined in reality.

In real world OTC markets, dealers rarely post full demand curves.[8] Instead, dealers engage in bilateral negotiations with their counterparties by quoting prices which are valid for a certain quantity. To capture this feature, we introduce a variant of the OTC game where dealers find the equilibrium prices and quantities through a sequence of bilateral exchange of quotes.

In particular, consider that each node in a given network is a trading desk. Each desk i consists of a desk-head, whom we continue to refer as dealer i, and one or more traders. Dealer i designs a bidding strategy and the traders have to implement this strategy. The bidding strategy describes how the traders should respond to bids they receive from counterparties. Bidding takes place sequentially, in rounds. In each bidding round τ, each trading desk i makes an offer $\pi^i_{ij,\tau} = \left\{p^i_{ij,\tau}, q^i_{ij,\tau}\right\}$ to each trading desk j with whom she has a link, indicating that she is willing to trade quantity $q^i_{ij,\tau}$ for price $p^i_{ij,\tau}$. Thus, in any bidding round all dealers make and receive offers to and from their counterparties in the network. If the price offered by i to j is arbitrarily close to the price offered by j to i, and the two quantities differ only to the extent of the order of the customers' demand at the given price, then the offer is accepted. Otherwise, a trading desk j that receives the bid in round τ, responds with a counter-offer $\pi^j_{ij,\tau+1} = \left\{p^j_{ij,\tau+1}, q^j_{ij,\tau+1}\right\}$ in round $\tau+1$. This process can continue for any number of rounds, until all trading desks accept the offers. At that point, trades are executed both across dealers and with customers.

More formally, in round 0, each dealer i chooses a bidding strategy $B^i_{ij}(s^i; \left\{\pi^j_{ij,\tau}\right\}_{j\in g^i})$ that describes the counter-offers that traders at desk i should make in round $\tau+1$, conditional on the bids they received in round $\tau \geq 0$, such that

$$B^i_{ij}(s^i; \left\{\pi^j_{ij,\tau}\right\}_{j\in g^i}) = \pi^i_{ij,\tau+1}, \tag{D.1}$$

for each $j \in g^i$. If there exists a price and quantity vector $\left\{\bar{p}^i_{ij}, \bar{q}^j_{ij}\right\}_{ij\in g}$ with

$$\bar{p}^i_{ij} = \bar{p}^j_{ij},$$
$$\bar{q}^i_{ij} + \bar{q}^j_{ij} + \beta_{ij}\bar{p}^i_{ij} = 0,$$

and

$$\lim_{\tau\to\infty} \pi^i_{ij,\tau} = \left(\bar{p}^i_{ij}, \bar{q}^i_{ij}\right),$$

[8] Duffie (2012*b*) describes a notable example where dealers do post full demand curves:

> In some active OTC derivative markets, such as the market for credit default swaps, clients of dealers can request "dealer runs," which are essentially lists of dealers' propsective bid and offer prices on a menu of potential trades.

for every $ij \in g$ and for any random starting vector $\left\{\pi^i_{ij,0}\right\}_{ij\in g}$, then trade takes place.

The pay-off for a dealer i is the expected profit $E\left[\sum_{j\in g^i} \bar{q}^i_{ij}\left(\theta^i - \bar{p}^i_{ij}\right)\right]$, provided $\left\{\bar{p}^i_{ij}, \bar{q}^j_{ij}\right\}_{ij\in g}$ exist, and minus infinity otherwise. Thus, taking each other dealers' bidding strategy as given, dealer i solves

$$\max_{\left\{B^i_{ij}(s^i;\left\{\pi^j_{ij,\tau}\right\}_{j\in g^i})\right\}_{j\in g^i}} E\left[\sum_{j\in g^i} \bar{q}^i_{ij}\left(\theta^i - \bar{p}^i_{ij}\right)|s^i\right].$$

This structure defines a game, which we refer to as the *price-discovery game.* The following proposition proves our claim that dealers can find the equilibrium prices and quantities in the OTC game by playing the price-discovery game.

Proposition D.2 *Suppose that there exists an equilibrium in the OTC game, when $\bar{Y} \geq 0$ and $\bar{Z} \geq 0$. Then:*

1. *There exists a set of bidding strategies $\left\{B^i_{ij}(s^i;\left\{\pi^j_{ij,\tau}\right\}_{j\in g^i})\right\}_{j\in g^i}$ that are an equilibrium in the price-discovery game.*
2. *The resulting prices and quantities $\left\{\bar{p}^i_{ij}, \bar{q}^i_{ij}\right\}_{ij\in g}$ are the same as the equilibrium prices and quantities in the OTC game.*

Proof. Starting from an equilibrium in the OTC game, we construct a bidding strategy for dealer i as follows. When a trader at desk i receives a bid $\pi^j_{ij,\tau} = \{p^j_{ij,\tau}, q^j_{ij,\tau}\}$ from each of his counterparties $j \in g^i$, she transforms $p^j_{ij,\tau}$ to

$$e^j_\tau = \frac{p^j_{ij,\tau}\left(t^i_{ij} + t^j_{ij} - \beta_{ij}\right) - t^i_{ij}e^i_{\tau-1}}{t^j_{ij}}$$

for each $j \in g^i$. Then, she updates her expectation about the asset value to be

$$e^i_{\tau+1} = \bar{y}^i s^i + \bar{\mathbf{z}}_{g^i}\mathbf{e}_{g^i,\tau}. \tag{D.2}$$

Finally, she constructs the counter-offer $\pi^i_{ij,\tau+1}$ with elements

$$\begin{aligned} p^i_{ij,\tau+1} &= \frac{t^i_{ij}e^i_{\tau+1} + t^j_{ij}e^j_\tau}{t^i_{ij} + t^j_{ij} - \beta_{ij}} \\ q^i_{ij,\tau+1} &= t^i_{ij}\left(e^i_{\tau+1} - p^i_{ij,\tau+1}\right). \end{aligned}$$

First, we show that if bidding functions are defined as above, the OTC price-discovery process converges to the equilibrium prices and quantities in the OTC game. To see this, we write (D.2) in matrix form as

$$\mathbf{e}_{\tau+1} = \bar{Y}\mathbf{s} + \bar{Z}\mathbf{e}_\tau.$$

where $\mathbf{e}_{\tau+1} = \left(e^i_{\tau+1}\right)_{i=1..n}$. Note that starting from any random vector $\mathbf{e}_0$ we'll have

$$\mathbf{e}_{\tau+1} = \left(I + \bar{Z} + ... + \left(\bar{Z}\right)^\tau\right)\bar{Y}\mathbf{s} + \left(\bar{Z}\right)^{\tau+1}\mathbf{e}_0.$$

In step 2 of the proof of Proposition 3, we show that the fact that all elements of $\bar{Z}$ are positive together with the existence of equilibrium in the conditional guessing game imply that $\lim_{u\to\infty}\left(\bar{Z}\right)^{u+1}=0$, which in turn implies that $\left(I-\bar{Z}\right)$ is nonsingular (see Meyer (2000) page 618), $\left(I-\bar{Z}\right)^{-1}\geq 0$ and

$$\left(I-\bar{Z}\right)^{-1}=\sum_{\tau=1}^{\infty}\left(\bar{Z}\right)^{\tau}.$$

(see Meyer (2000) pp. 620 & 618.)

Thus, we have that

$$\lim_{\tau\to\infty}\mathbf{e}_{\tau+1}=\left(I-\bar{Z}\right)^{-1}\bar{Y}\mathbf{s},$$

or the equilibrium expectations in the OTC game. But then, by definition, $\left\{\bar{p}_{ij}^{i},\bar{q}_{ij}^{i}\right\}_{ij\in g}$ exist and coincide with the equilibrium of the OTC game.

The last step is to show that dealer i would not want to change her bidding strategy unilaterally. Note that for any such deviation to be meaningful, it has to imply alternative limit price and quantity vectors. If there is no convergence, dealer i receives a payoff of minus infinity. However, by construction, if a modified bidding strategy converges to different price and quantity vectors, then these vectors are also fixed points of generalized demand curves in the OTC game. However, the other dealers' bidding strategies are constructed based on their equilibrium demand functions in the OTC game. This implies that if dealer i wants to deviate from the equilibrium bidding strategies in the price-discovery game, he wants to deviate from his generalized demand curve in the original OTC game as well. But this is a contradiction. ■

While the construction of the price-discovery game is arguably artificial, it illustrates some important features of our OTC game. On the one hand, finding the equilibrium prices and quantities in the OTC game needs not rely on any kind of auctioneer. The price-discovery game shows that equilibrium prices and quantities can be found by an iterative, decentralized process. Moreover, the coefficients of the generalized demand curves (and, consequently the coefficients of the bidding strategies) can be derived without observing the realization of signals. Indeed, dealers can find the equilibrium coefficients just by understanding the structure of the game they are facing.

On the other hand, the price discovery game emphasizes an important limitation of the OTC game. Namely, the equivalence between the price discovery game and the OTC game relies on the fact that the bidding functions $B_{ij}^{i}(\cdot)$ are static. That is, they do not depend on the bidding round, τ, and they are conditional only on the outcome of the last bidding round $\pi_{i,\tau}^{j}$, as opposed to all previous rounds. This restriction on the strategy space of dealers that dealers can use is necessary as the OTC game is nevertheless a static game. Thus, we are not able to capture in our framework any dynamic learning considerations of real world OTC markets.

E Appendix: Simulation details and further figures

In this Appendix, we provide details and further results on our simulation exercise in Section 6.

E.1 Network parameterization and calibration

For generating our random networks and for the calibration of the baseline case, we closely follow Jackson and Rogers (2007). In particular, the network formation starts with a small complete network. Then, nodes are born sequentially. When a new node is born, m_R parents are randomly chosen from the existing nodes with uniform probability. The new node forms a link with any given parent with probability p. Then, from the set of existing connections of the parents m_S nodes are chosen randomly. The new node forms links with any given of these nodes with probability p. Hence, in expectation, each node forms $m \equiv p\,(m_S + m_R)$ connections where the ratio of randomly selected to network based nodes is $r \equiv \frac{m_R}{m_S}$. To make this process well defined the starting complete network has $m_R + m_S + 1$ nodes. As Jackson and Rogers (2007) shows that for large networks the cumulative degree distribution function is

$$F\,(d) = 1 - \left(\frac{rm}{d + rm}\right)^{1+r}$$

where d is the number of degrees. When r is small, the resulting networks show core-periphery features, similarly to the preferential attachment model, while when r is very large, the resulting network is close to a random network where each link is formed with equal probability.

For our baseline calibration, we use the analysis in Brunnermeier, Clerc and Scheicher (2013) on the European CDS market. In particular, to keep our exercise computationally manageable, we use the 10x10x10 representation of this market on their Figure 8. This representation reduces the total network of more than thousand nodes to a network of 54 dealers following Duffie (2011)'s proposal by focusing on the largest positions of the largest traders in the most important assets. Following the method of Jackson and Rogers (2007), we set m to the average degree in this network, calibrate r to the shape of the degree distribution and set p to fit the average clustering coefficient. This process gives us the baseline parameters of $r = \frac{1}{3}, m_R = 2, m_S = 6, p = 0.3$.

Given the baseline parameters for the network formation process we generate 50 random networks of 54 dealers. With each of this networks, and with the baseline informational parameters $\rho = 0.5, \sigma_\theta^2 = \sigma_\varepsilon^2 = 1$, we calculate our market-wide measures of price dispersion, concentration of trade and informational efficiency. We also run 5 single variable regressions (with a constant) with the degree centrality of a given dealer on the right hand side and the expected profit of a dealer, the expected intermediation, the average cost of trade for a dealer, the expected gross volume and the reduction in conditional variance on the left hand side. We record the slope coefficients in these regressions. Considering the 50 estimates for each slopes and market-wide measures, we can interpret the means of each 50 estimates as our point-estimate for the given network parametrization and the range of the coefficients as simulated confidence intervals around that point estimate. For example, dropping the maximal and the minimal elements from each 50 estimates for a given measure we would get the confidence intervals of 96% percent. For transparency, we do not drop any slope estimates, but plot each.

As a sensitivity analysis for the informational parameters, we recalculate our market-wide measures rerun these regressions for different values of ρ, and the noise-to-signal ratio, $\sigma^2 = \frac{\sigma_\varepsilon^2}{\sigma_\theta^2}$. For each set of parameters we redraw the 50 networks. We report only the results on varying σ^2 in the main text. Our results on varying ρ is in this Online Appendix.

As a sensitivity analysis for the shape of the network, we generate random networks with different r values and different p values keeping the size of the network and the average number of links, m, fixed.. In particular, first we keep $p = 0.3$, but consider the pairs $(2,6), (3,5), (4,4), (5,3), (6,2)$ for (m_R, m_S). Just as before, for each set of parameters, we generate 50 random networks, re-run our regressions and recalculate our market-wide measures. We keep the informational parameters fixed at their baseline values. This is the parameterization we explore in the main text. Then, we consider $p = 0.6$ with the pairs $(1,3), (2,2), (3,1)$ for (m_R, m_S). Again, for each set of parameters, we generate 50 random networks, re-run our regressions and recalculate our market-wide measures. In this way, we obtain point-estimates of each of the slopes and market-wide measures for each parametrization of the network together with simulated standard errors. Our results with $p = 0.6$ is in the next subsection.

In the rest of this Appendix, we return to the database generated by our baseline simulation, but plot the results differently. Instead of putting the parameter r on the x-axis (a parameter which was an input for the random network generation and which we varied exogenously), we are putting realized network characteristics on the x-axis as clustering, assortativity and diameter. For easier interpretation, we have also put a regression line into each of the panels. We leave the detailed analysis of these figures to the reader.

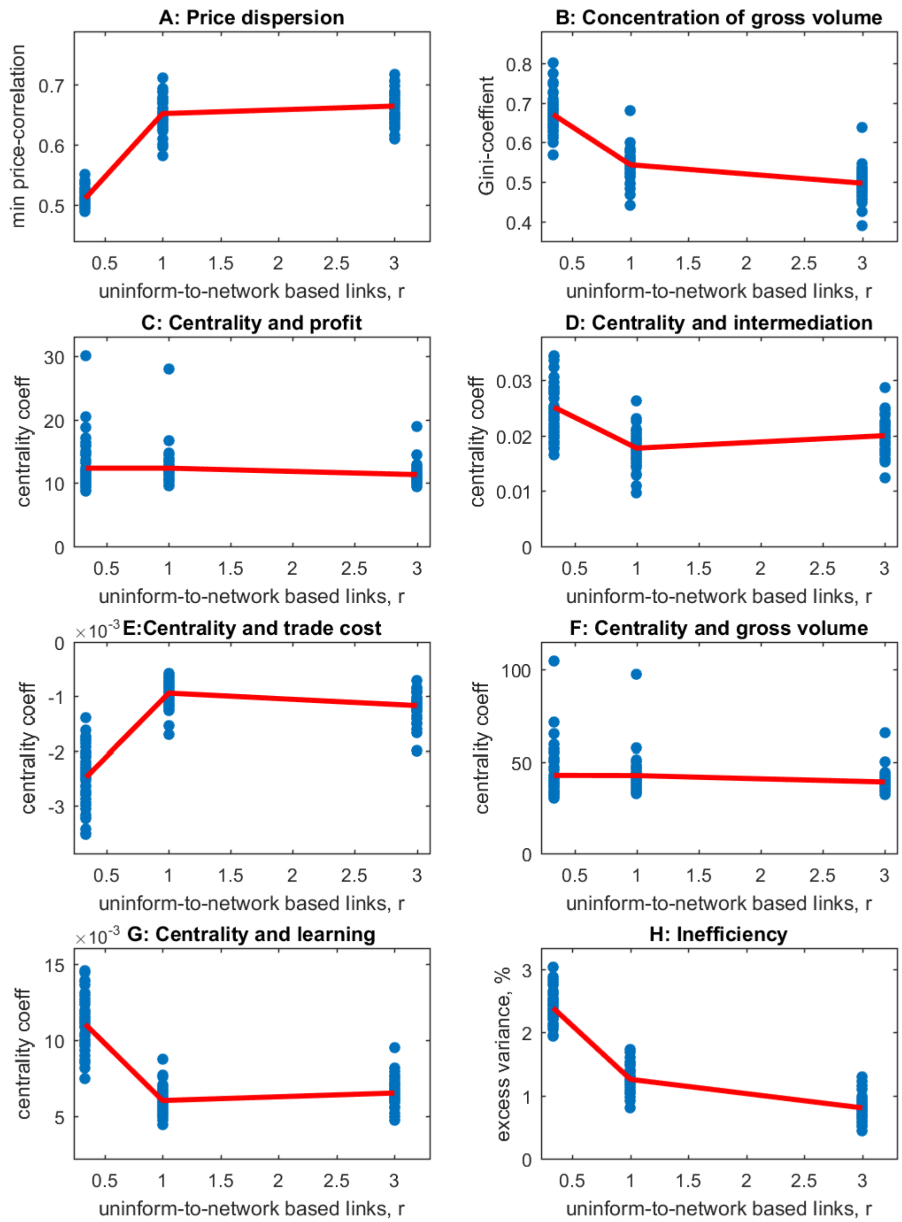


Figure E.1: Panels A,B and H show the minimal element in the price correlation matrix, the Gini-coefficient of volume and excess variance compared to the planner's solution, while C-G show slope coefficients in regressions of expected profit, expected intermediation, average price impact, gross volume and learning on centrality on simulated networks. Each random network is generated by the method in Jackson and Rogers (2007) with varying uninform-to-network based links ratio. For a given r, 50 networks are generated. Each blue circle corresponds to a network realization, while the red line shows the average for a set of networks with fixed parameters. Parameters are $n = 54$ $m = 8$, $p = 0.6$. $\sigma_\varepsilon^2 = \sigma_\theta^2, \rho = 0.5$.

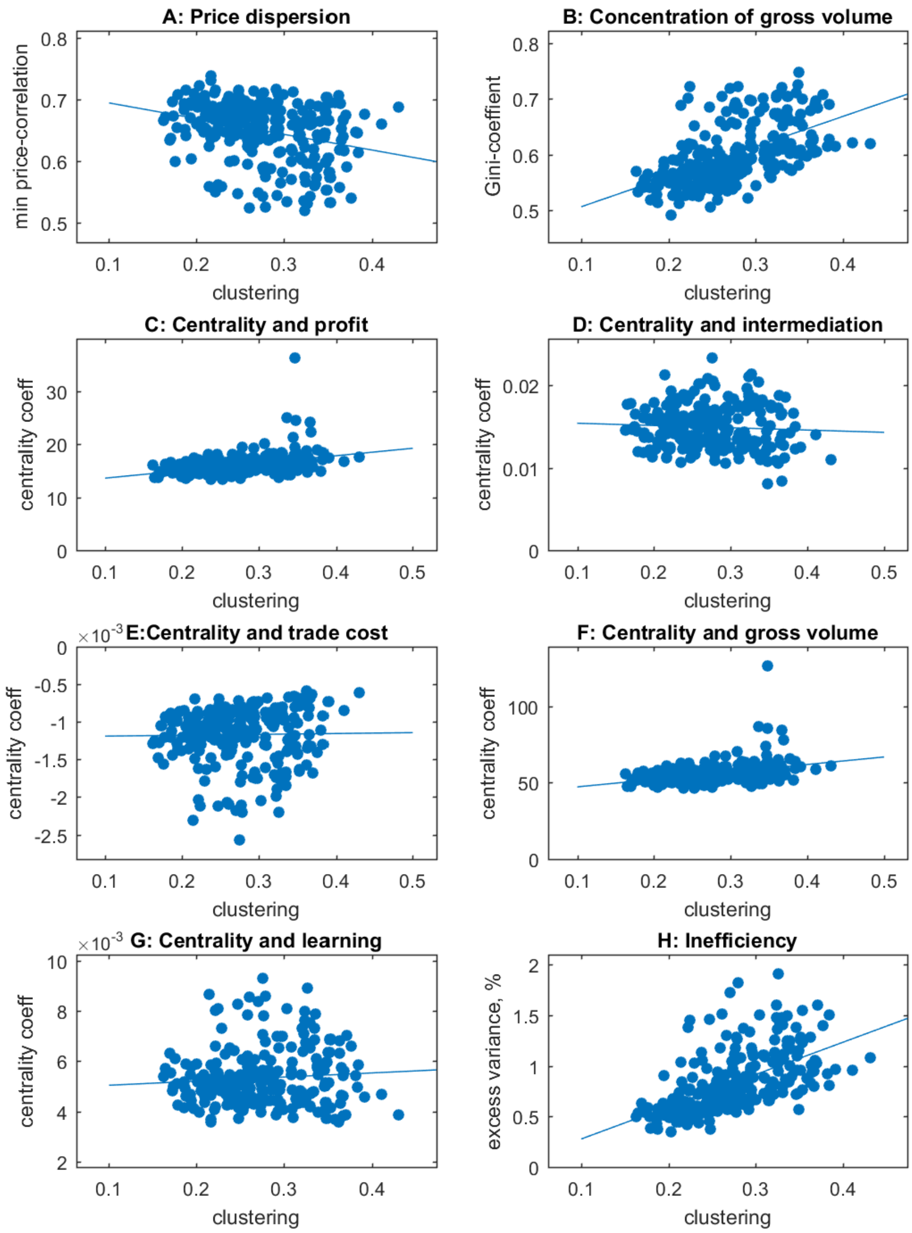


Figure E.2: Panels A,B and H show the minimal element in the price correlation matrix, the Gini-coefficient of volume and excess variance compared to the planner's solution, while C-G show slope coefficients in regressions of expected profit, expected intermediation, average price impact, gross volume and learning on centrality on simulated networks. Each random network is generated by the method in Jackson and Rogers (2007) with varying uninform-to-network based links ratio. For a given r, 50 networks are generated. Each blue circle corresponds to a network realization, while the red line shows the average for a set of networks with fixed parameters. We plot the results as a function of the clustering coefficient in the underlying network. Parameters are $n = 54$ $m = 8$, $p = 0.3$. $\sigma_\varepsilon^2 = \sigma_\theta^2, \rho = 0.5$.

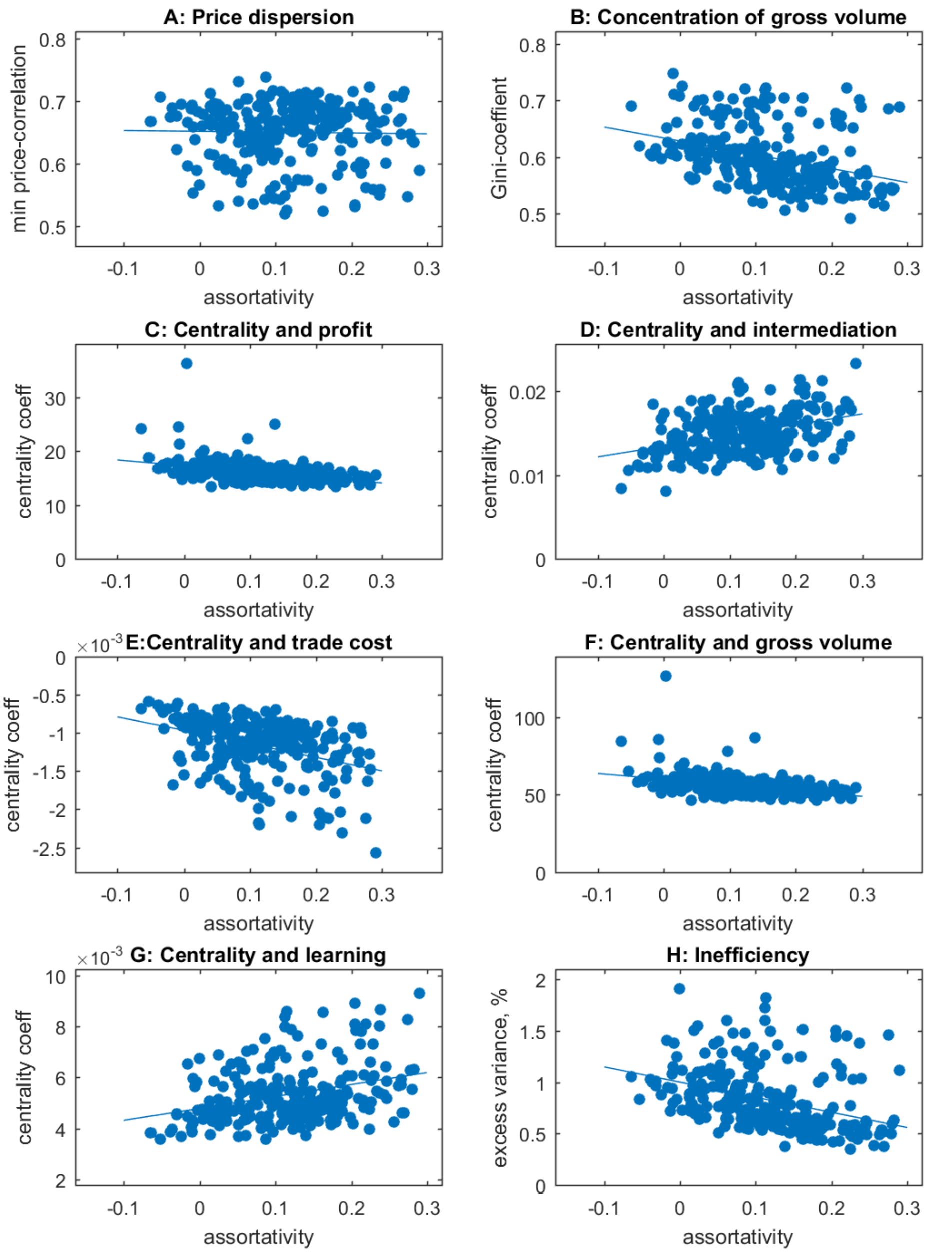


Figure E.3: Panels A,B and H show the minimal element in the price correlation matrix, the Gini-coefficient of volume and excess variance compared to the planner's solution, while C-G show slope coefficients in regressions of expected profit, expected intermediation, average price impact, gross volume and learning on centrality on simulated networks. Each random network is generated by the method in Jackson and Rogers (2007) with varying uninform-to-network based links ratio. For a given r, 50 networks are generated. Each blue circle corresponds to a network realization, while the red line shows the average for a set of networks with fixed parameters. We plot the results as a function of the assortativity coefficient in the underlying network. Parameters are $n = 54$ $m = 8$, $p = 0.3$. $\sigma_\varepsilon^2 = \sigma_\theta^2, \rho = 0.5$.

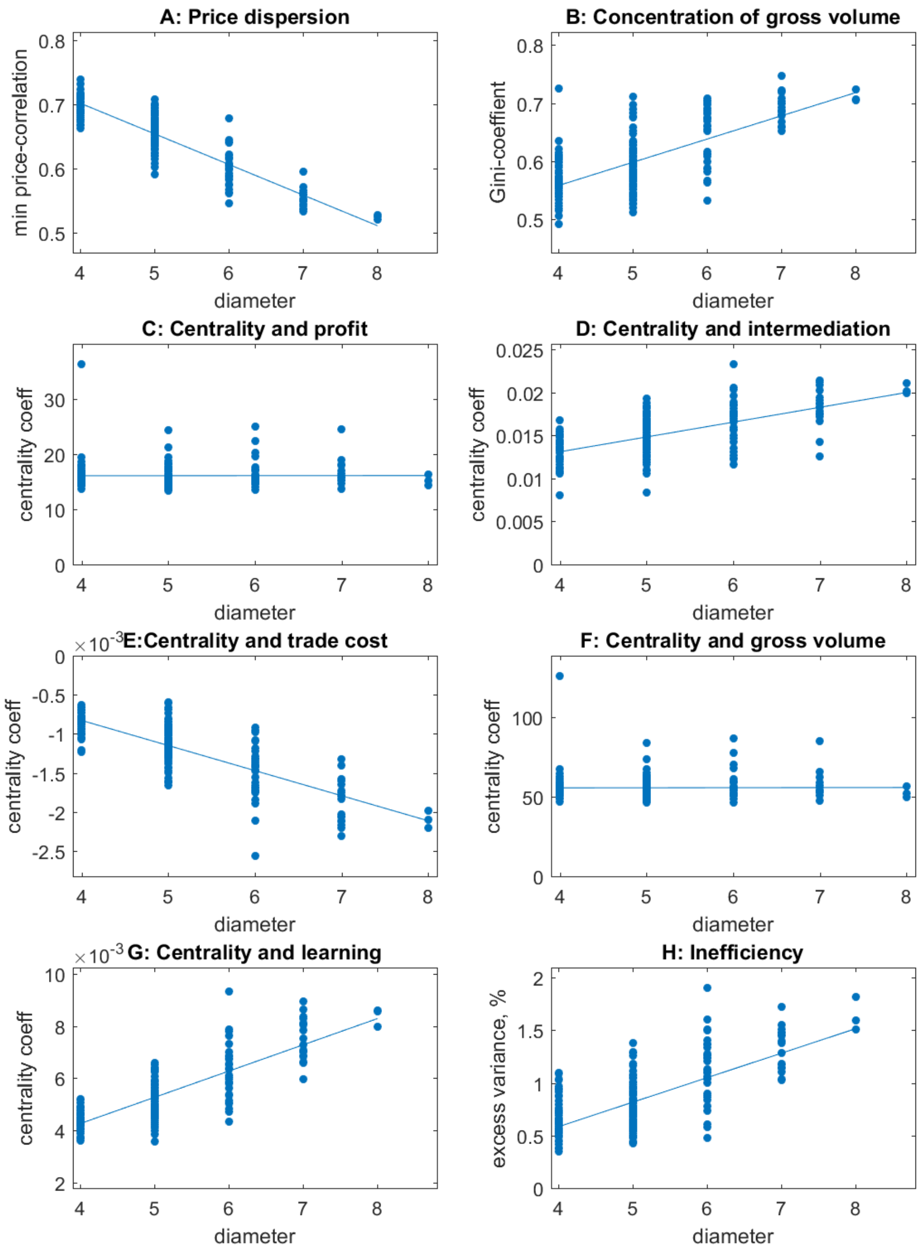


Figure E.4: Panels A,B and H show the minimal element in the price correlation matrix, the Gini-coefficient of volume and excess variance compared to the planner's solution, while C-G show slope coefficients in regressions of expected profit, expected intermediation, average price impact, gross volume and learning on centrality on simulated networks. Each random network is generated by the method in Jackson and Rogers (2007) with varying uninform-to-network based links ratio. For a given r, 50 networks are generated. Each blue circle corresponds to a network realization, while the red line shows the average for a set of networks with fixed parameters. We plot the results as a function of the diameter in the underlying network. Parameters are $n = 54$ $m = 8$, $p = 0.3$. $\sigma_\varepsilon^2 = \sigma_\theta^2, \rho = 0.5$.

F Appendix: When does the equilibrium of the OTC game exists?

We showed in Proposition 2 that an equilibrium exists when the solution, z^i_{ij}, of the system (21) is in the interval $(0, 2)$. As Section 6 illustrates, apart from the networks characterized in Proposition 3, we found that the equilibrium exists for a large range of parameters for a wide range of relevant random networks. However, there exist parameters for which in certain networks the conditions of Proposition 2 are not satisfied.

In all the examples we have found, there is at least one agent who puts negative weight on at least one of her neighbors expectation in the conditional game, that is, $\bar{z}^i_{ij} < 0$ for some i and ij. This is possible as the correlation between θ^i and e^j, conditional on all the other expectations of i's neighbors and s^i might be negative. While this is still a valid equilibrium of the conditional guessing game, it results in a negative z^i_{ij} in the OTC game, which violates the second-order conditions.

While we do not have a proof for the following conjecture, we believe that for any network and any parameters for which all $\bar{z}^i_{ij} \geq 0$ in the equilibrium of the conditional guessing game, the implied z^i_{ij} are in the interval $(0, 2)$, therefore the equilibrium of the OTC game exists. Given that the first step in the first part of Proof of Proposition 3 implies that for any network, if ρ sufficiently small, then $\bar{z}^i_{ij} \geq 0$, this conjecture, if true, would imply by continuity that there is an equilibrium of the OTC game for any network as long as ρ is smaller than a given threshold.

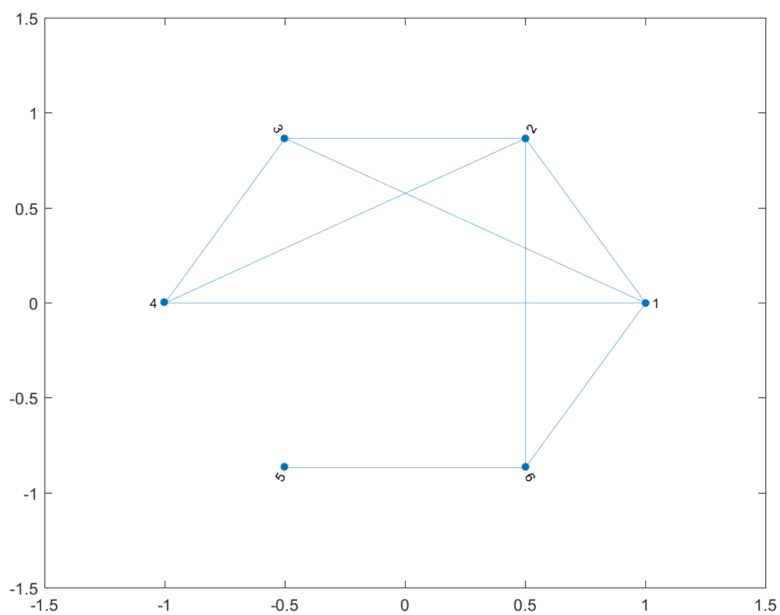


Figure F.5: An example of a graph implying negative $\bar{z}^1_{12}$,$\bar{z}^2_{21}$ and z_{12},z_{21} values when $\rho = 0.9$.

In any case, Figure F.5 shows an example of a graph for which we do not find an equilibrium of the OTC game when $\rho = 0.9$.(or larger). In particular, in the conditional guessing game $\bar{z}^1_{12}$ and $\bar{z}^2_{21}$ are negative implying that the equation system in Proposition 2 implies negative solutions for z_{12}^1 and z_{21}^2 too. For the intuition, observe that 1, 2,3 and 4 are all connected to each other, but only 1 and 2 are connected to 6 who is the only one to be connected to 5. To separate the signal of 5, 1 would subtract the posterior of 2 from the posterior of 6. As the posteriors of 3 and 4 has about the same information on the common value element than 2, the best use of the posterior of 2 is to separate the signal of 5 this way. This, and the symmetric argument that 2 subtracts the posterior of 1 from the posterior of 5, explains why $\bar{z}_{12}^1$ and $\bar{z}_{21}^2$ are negative. When ρ is smaller, 3 and 4 learn less about the common value element so 1 uses the posterior of 2 both for guessing the common value element (implying a positive component in $\bar{z}_{12}^1$) and for the separation of signal 6. When ρ is sufficiently small for the first effect to dominate, the existence of the equilibrium is restored.