# Asset Commonality, Debt Maturity and Systemic Risk*

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**Abstract**

We develop a model in which asset commonality and short-term debt of banks interact to generate excessive systemic risk. Banks swap assets to diversify their individual risk. Two asset structures arise. In a clustered structure, groups of banks hold common asset portfolios and default together. In an unclustered structure, defaults are more dispersed. Portfolio quality of individual banks is opaque but can be inferred by creditors from aggregate signals about bank solvency. When bank debt is short-term, creditors do not roll over in response to adverse signals and all banks are inefficiently liquidated. This information contagion is more likely under clustered asset structures. In contrast, when bank debt is long-term, welfare is the same under both asset structures.

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# 1 Introduction

Understanding the nature of systemic risk is key to understanding the occurrence and propagation of financial crises. Traditionally the term "systemic risk" describes a situation where many (if not all) financial institutions fail as a result of a common shock or a contagion process. A typical common shock leading to systemic failures is a collapse of residential or commercial real estate values (see Reinhart and Rogoff, 2009). Contagion refers to the risk that the failure of one financial institution leads to the default of others through a domino effect in the interbank market, the payment system or though asset prices (see, for example, the survey in Allen, Babus and Carletti, 2009).

The recent developments in financial markets and the crisis that started in 2007 have highlighted the importance of another type of systemic risk related to the linkages among financial institutions and to their funding maturity. The emergence of financial instruments in the form of credit default swaps and similar products has improved the possibility for financial institutions to diversify risk, but it has also increased the overlaps in their portfolios. Whether and how such asset commonality among banks leads to systemic risk may depend on their funding maturity structure. With short-term debt, banks are informationally linked. Investors respond to the arrival of interim information in a way that depends on the composition of their asset structures. With long-term debt instead, interim information plays no role and the composition of asset structures does not matter for systemic risk.

In this paper we analyze the interaction between asset commonality and funding maturity in generating systemic risk through an informational channel. We develop a simple two-period model, where each bank issues debt to finance a risky project. We initially consider the case of long-term debt and then that of short-term debt. Projects are risky and thus banks may default at the final date. Bankruptcy is costly in that investors only recover a fraction of the bank's project return. As project returns are independently distributed, banks have an incentive to diversify to lower their individual default probability. We model this by assuming that each bank can exchange shares of its own project with other banks. Exchanging projects is costly as it entails a due diligence cost for each swapped project. In equilibrium, banks trade off the advantages of diversification in terms of lower default probability with the due diligence costs.[1]

Swapping projects can generate different types of overlaps in banks' portfolios. We model banks' portfolio decisions as a network formation game, where banks choose the number of projects to exchange but cannot coordinate on the composition of their asset structures. For ease of exposition, we focus on the case of six banks with each of them optimally exchanging projects with two other banks. This leads to two possible asset structures. In one, which we call "clustered", banks are connected in two clusters of three banks each. Within each cluster all banks hold the same portfolio, but the two clusters are independent of each other. In the second, which we call "unclustered", banks are connected in a circle. Each of them swaps projects only with the two neighboring banks and none of the banks holds identical portfolios.

We show that with long-term debt the asset structure does not matter for welfare. The reason is that in either structure each bank's portfolio is formed by three independently distributed projects with the same distribution of returns. The number of bank defaults and the expected costs of default are the same in the two structures and so is total welfare.

In contrast, the asset structure plays an important role in determining systemic risk and welfare when banks use short-term debt. The main difference is that at the intermediate date investors receive a signal concerning banks' future solvency. The signal indicates whether all banks will be solvent in the final period (good news) or whether at least one of them will default (bad news). The idea is that banks' assets are opaque (see, e.g., Morgan, 2004; Flannery, Kwan and Nimalendran, 2010) and thus the market receives information on banks' overall solvency rather than on the precise value of banks' asset fundamental values. Upon observing the signal, investors update the probability that their bank will

[1] The assumption that exchanging projects entails a due diligence cost implies that banks do not find it optimal to fully diversify. There are other ways to obtain limited diversification. For example, a decreasing marginal benefit of diversification or an increasing marginal cost would lead to the same result.

be solvent at the final date and roll over the debt if they expect to be able to recover their opportunity cost. Rollover always occurs after a good signal is realized but not after a bad signal arrives. When rollover does not occur, all banks are forced into early liquidation. The failure to roll over is the source of systemic risk in our analysis.

Investors' rollover decisions depend on the structure of asset overlaps, the opportunity cost and the bankruptcy cost. We show that, upon the arrival of bad news, rollover occurs less often in the clustered than in the unclustered asset structure. When investors recover enough in bankruptcy or have a low opportunity cost, debt is rolled over in both structures. As the amount they recover decreases and their opportunity cost increases, debt is still rolled over in the unclustered structure but not in the clustered one. The reason is that there is a greater information spillover in the latter as defaults are more concentrated. Upon the arrival of negative information investors infer that the conditional default probability is high and thus decide not to roll over. In the unclustered structure defaults are less concentrated and the arrival of the bad signal indicates a lower probability of a rash of bank defaults. When investors obtain little after banks default because of high bankruptcy costs or have a high opportunity cost, banks are liquidated early in both structures.

Even if the clustered structure entails more rollover risk than the unclustered structure, it does not always lead to lower welfare. The optimal asset structure with short-term finance depends on investors' rollover decisions, the proceeds from early liquidation and the bankruptcy costs. When banks continue and offer investors a repayment of the same magnitude in either structure, total welfare is the same in both structures. When the debt rollover requires a higher promised repayment in the clustered than in the unclustered structure, welfare is higher in the latter as it entails lower bankruptcy costs. When banks are liquidated early in the clustered structure only, the comparison of total welfare becomes ambiguous. In the arguably more plausible case when neither the bankruptcy costs nor the proceeds from early liquidation are too high, total welfare remains higher in the unclustered structure. When instead investors recover little after bankruptcy and obtain large proceeds from early liquidation, welfare becomes higher in the clustered structure, and remains so even when early liquidation occurs in both structures.[2]

To summarize, the paper shows that clustered asset structures entail higher systemic risk when bad information about banks' future solvency arrives in the economy. This implies that unclustered asset structures typically lead to higher welfare, although there are cases where clustered structures can be superior. The focus of the analysis is the interaction of banks' asset structures, information and debt maturity in generating systemic risk. The crucial point is that the use of short-term debt may lead to information contagion among financial institutions. The extent to which this happens depends on the composition of the asset structure, that is on the degree of overlap of banks' portfolios. This result raises the question of why banks use short-term debt in the first place. We show that the optimality of short-term debt depends on the asset structure and on the difference between the long-term and the short-term rate that investors can obtain from alternative investments. The market failure in our model is that banks are unable to coordinate on a particular composition of asset structure. By choosing the efficient maturity of the debt they can improve their expected profits and welfare, but cannot ensure the emergence of the optimal asset structure.

Our paper is related to several strands of literature. Concerning the effects of diversification on banks' portfolio risk, Shaffer (1994), Wagner (2010) and Ibragimov, Jaffee and Walden (2010) show that diversification is good for each bank individually, but it can lead to greater systemic risk as banks' investments become more similar. As a consequence, it may be optimal to limit diversification.

Other papers analyze the rollover risk entailed in short-term finance. Acharya, Gale and Yorulmazer (2010) and He and Xiong (2009) show that rollover risk can lead to market freezes and dynamic bank runs. Diamond and Rajan (2010) and Bolton, Santos

[2] This latter case is presumably less plausible. An example would be where the project has a high resale value because of the possibility of many alternative uses of its equipment in the first period, but low proceeds in the second period because of high direct and indirect bankruptcy costs.

and Scheinkman (2010) analyze how liquidity dry-ups can arise from the fear of fire sales or asymmetric information. All these studies use a representative bank/agent framework. By contrast, we analyze a framework with multiple banks and show how different asset structures affect the rollover risk resulting from short-term finance.

Systemic risk arises in our model from the investors' response to the arrival of interim information regarding banks' future solvency. In this sense our paper is related to the literature on information contagion. Chen (1999) shows that sufficient negative information on the number of banks failing in the economy can generate widespread runs among depositors at other banks whose returns depend on some common factors. Dasgupta (2004) shows that linkages between banks in the form of deposit crossholdings can be a source of contagion when the arrival of negative interim information leads to coordination problems among depositors and widespread runs. Acharya and Yorulmazer (2008) find that banks herd and undertake correlated investment to minimize the effect of information contagion on the expected cost of borrowing. Our paper also analyzes the systemic risk stemming from multiple structures of asset commonality among banks, but it focuses on the interaction with the funding maturity of financial intermediaries.

Some other papers study the extent to which banks internalize the negative externalities that arise from contagion. Babus (2009) proposes a model where banks share the risk that the failure of one bank propagates through contagion to the entire system. Castiglionesi and Navarro (2010) show that an agency problem between bank shareholders and debtholders leads to fragile financial networks. Zawadowski (2010) argues that banks that are connected in a network of hedging contracts fail to internalize the negative effect of their own failure. All these papers rely on a domino effect as a source of systemic risk. In contrast, we focus on asset commonality as a source of systemic risk in the presence of information externalities when banks use short-term debt.

The rest of the paper proceeds as follows. Section 2 lays out the basic model when banks use long-term debt. Section 3 describes the equilibrium that emerges with long-term finance. Section 4 introduces short-term debt. It analyzes investors' decision to roll over the debt in response to information about banks' future solvency and the welfare properties of the different asset structures. Section 5 discusses a number of extensions. Section 6 concludes.

## 2 The basic model with long-term finance

Consider a three-date ($t = 0, 1, 2$) economy with six risk-neutral banks, denoted by $i = 1, ..., 6$, and a continuum of small, risk-neutral investors. Each bank $i$ has access at date 0 to an investment project that yields a stochastic return $\theta_i = \{R_H, R_L\}$ at date 2 with probability $p$ and $1 - p$, respectively, and $R_H > R_L > 0$. The returns of the projects are independently distributed across banks.

Banks raise one unit of funds each from investors at date 0 and offer them, in exchange, a long-term debt contract that specifies an interest rate $r$ to be paid at date 2. Investors provide finance to one bank only and are willing to do so if they expect to recover at least their two-period opportunity cost $r_F^2 < E(\theta_i)$.

We assume that $R_H > r_F^2 > R_L$ so that a bank can pay $r$ only when the project yields a high return. When the project yields a low return $R_L$, the bank defaults at date 2 and investors recover a fraction $\alpha \in [0, 1]$ of the project return. The remaining fraction $(1 - \alpha)$ is lost as bankruptcy costs. Thus, investors will finance the bank only if their participation constraint

$$pr + (1 - p)\alpha R_L \geq r_F^2 \tag{1}$$

is satisfied. The first term on the left hand side represents the expected payoff to the investors when the bank repays them in full. The second term represents investors' expected payoff when the bank defaults at date 2. The right hand side is the investors' opportunity cost.

When the project returns $R_H$, the bank acquires the surplus $(R_H - r)$. Otherwise, it receives 0. The bank's expected profit is then given by

$$\pi_i = p(R_H - r). \tag{2}$$

Given projects are risky and returns are independently distributed, banks can reduce their default risk through diversification. We model this by assuming that each bank can exchange shares of its own project with $\ell_i$ other banks through bilateral connections. That is, bank $i$ exchanges a share of its project with bank $j$ if and only if bank $j$ exchanges a share of its project with bank $i$. A bilateral swap of projects creates a link $\ell_{ij}$ between banks $i$ and $j$. Then each bank $i$ ends up with a portfolio of $1+\ell_i$ projects with a return equal to

$$X_i = \frac{\theta_{i_1} + \theta_{i_2} + ... + \theta_{i_{1+\ell_i}}}{1+\ell_i}. \tag{3}$$

The exchange of project shares creates linkages and portfolio overlaps among banks as each of them has shares of $1+\ell_i$ independently distributed projects in its portfolio. The collection of all linkages can be described as an asset structure $g$. The degree of overlaps in banks' portfolios depends on the number $\ell_i$ of projects that each bank swaps with other banks and on the composition of banks' asset structures.

Exchanging projects with other banks reduces the expected bankruptcy costs $(1-p)(1-\alpha)R_L$ and investors' promised repayment $r$ but it also entails a due diligence cost $c$ per link. The idea is that banks know their own project, but they do not know those of the other banks. Thus they need to exert costly effort to check that the projects of the other banks are bona fide as well. This limits the benefits of diversification and allows us to focus on a situation where banks do not perfectly diversify. In choosing the number of projects they wish to exchange, banks weigh the benefit of diversification in terms of lower bankruptcy costs against the increased due diligence costs.

## 3 Long-term finance

We model banks' portfolio decisions as a network formation game. This allows us to focus on the various asset structure compositions that emerge from the swapping of projects. We first derive the participation constraint of the investors and banks' profits when each bank $i$ has $\ell_i$ links with other banks and holds a portfolio of $1+\ell_i$ projects. An equilibrium asset structure is one where banks maximize their expected profits and do not find it worthwhile to sever or add a link.

We denote as $r \equiv r(g)$ the interest rate that bank $i$ promises investors in an asset structure $g$. Investors receive $r$ at date 2 when the return of bank $i$'s portfolio is $X_i \geq r$, while they receive a fraction $\alpha$ of the bank's portfolio return when $X_i < r$. The participation constraint of the investors is then given by

$$\Pr(X_i \geq r)r + \alpha E(X_i < r) \geq r_F^2, \tag{4}$$

where $\Pr(X_i \geq r)$ is the probability that the bank remains solvent at date 2 and $E(X_i < r) = \sum_{x<r} x \Pr(X_i = x)$ is the bank's expected portfolio payoff when it defaults at date 2. The equilibrium $r$ is the lowest interest rate that satisfies (4) with equality.

Banks receive the surplus $X_i - r$ whenever $X_i \geq r$ and 0 otherwise. The expected profit of a bank $i$ in an asset structure $g$ is

$$\pi_i(g) = E(X_i \geq r) - \Pr(X_i \geq r)r - c\ell_i, \tag{5}$$

where $E(X_i \geq r) = \sum_{x \geq r} x \Pr(X_i = x)$ is the expected return of the bank's portfolio and $\Pr(X_i \geq r)r$ is the expected repayment to investors when the bank remains solvent at date 2, and $c\ell_i$ are the total due diligence costs. Substituting the equilibrium interest rate $r$ from (4) with equality into (5), the expected profit of bank $i$ becomes

$$\pi_i(g) = E(X_i) - r_F^2 - (1-\alpha)E(X_i < r) - c\ell_i. \tag{6}$$

The bank's expected profit is given by the expected return of its portfolio $E(X_i)$ minus the investors' opportunity cost $r_F^2$, the expected bankruptcy costs $(1-\alpha)E(X_i < r)$, and the total due diligence costs $c\ell_i$. As (6) shows, greater diversification involves a trade-off between lower bankruptcy costs and higher total due diligence costs.

Banks choose the number of project shares to exchange $\ell_i$ in order to maximize their expected profits. The choice of $\ell_i$ determines the (possibly multiple) equilibrium asset structure(s). An asset structure $g$ is an equilibrium if it satisfies the notion of *pairwise stability* introduced by Jackson and Wolinsky (1996). This is defined as follows.

**Definition 1** *An asset structure $g$ is pairwise stable if*

*(i) for any pair of banks $i$ and $j$ that are linked in the asset structure $g$, neither of them has an incentive to unilaterally sever their link $\ell_{ij}$. That is, the expected profit each of them receives from deviating to the asset structure $(g - \ell_{ij})$ is not larger than the expected profit that each of them obtains in the asset structure $g$ $(\pi_i(g - \ell_{ij}) \leq \pi_i(g)$ and $\pi_j(g - \ell_{ij}) \leq \pi_j(g))$;*

*(ii) for any two banks $i$ and $j$ that are not linked in the asset structure $g$, at least one of them has no incentive to form the link $\ell_{ij}$. That is, the expected profit that at least one of them receives from deviating to the asset structure $(g + \ell_{ij})$ is not larger than the expected profit that it obtains in the asset structure $g$ $(\pi_i(g + \ell_{ij}) \leq \pi_i(g)$ and/or $\pi_j(g + \ell_{ij}) \leq \pi_j(g))$.*

To make the analysis more tractable, we impose a condition to ensure that for any $\ell_i = 0, .., 5$ the bank defaults and is unable to repay $r$ to investors at date 2 only when all projects in its portfolio pay off $R_L$. Thus, we can write the bank's default probability as $\Pr(X_i < r) = (1-p)^{1+\ell_i}$ and the probability of the bank being solvent at date 2 as $\Pr(X_i \geq r) = 1 - (1-p)^{1+\ell_i}$. To have this, it is sufficient to impose that the left hand side of (4) is decreasing in $\ell_i$ for any $\ell_i = 0, .., 5$[3] and that

$$(1-(1-p)^6)\frac{5R_L + R_H}{6} + (1-p)^6 \alpha R_L \geq r_F^2. \tag{7}$$

These conditions guarantee that there exists an interest rate $r$ in the interval $[r_F^2, \frac{\ell_i R_L + R_H}{1+\ell_i}]$ that satisfies the investors' participation constraint (4) for any $\ell_i = 0, .., 5$, where $\frac{\ell_i R_L + R_H}{1+\ell_i}$ is the next smallest return realization of a bank's portfolio after all projects return $R_L$.

Given (7), the bank's expected profit in (6) can be written as

$$\pi_i(g) = E(X_i) - r_F^2 - (1-p)^{1+\ell_i}(1-\alpha)R_L - c\ell_i. \tag{8}$$

It is easy to show that (8) is concave in $\ell_i$ as the second derivative with respect to $\ell_i$ is negative.

In what follows we will concentrate on the case where in equilibrium banks find it optimal to exchange $\ell_i = 2$ project shares and only symmetric asset structures are formed so that $\ell_i = \ell_j = \ell$. The reason is that this is the minimum number of links such that there are multiple nontrivial asset structures. We have the following.

**Proposition 1** *For any $c \in [p(1-p)^3(1-\alpha)R_L, p(1-p)^2(1-\alpha)R_L]$, a structure $g^*$ where all banks have $\ell^* = 2$ links is pairwise stable and Pareto dominates equilibria with $\ell^* \neq 2$.*

**Proof.** See the Appendix. ■

In equilibrium banks trade off the benefit of greater diversification in terms of lower expected bankruptcy costs with higher total due diligence costs. Proposition 1 identifies the parameter space for the cost $c$ such that this trade off is optimal at $\ell^* = 2$.

Banks choose the number of projects to exchange but not the composition of the asset structure so that multiple structures can emerge, for a given $\ell^*$. With $\ell^* = 2$ there are two equilibrium asset structures $g^*$ as shown in Fig. 1. In the first structure, which we define

[3] The condition guaranteeing that (4) is decreasing in $\ell_i$ is provided in Appendix A of Allen, Babus and Carletti (2011).

as "clustered" ($g = C$), banks are connected in two clusters of three banks each. Within each cluster, banks hold identical portfolios but the two clusters are independent of each other. In the second structure, denoted as "unclustered" ($g = U$), banks are all connected in a circle. Each of them exchanges projects only with the two neighboring banks so that none of the banks holds identical portfolios. In this sense, risk is more concentrated in the clustered than in the unclustered structure.

Both asset structures are pairwise stable if the due diligence cost $c$ is in the interval $[p(1-p)^3(1-\alpha)R_L, p(1-p)^2(1-\alpha)R_L]$. No bank has an incentive to deviate by severing or adding a link as it obtains higher expected profit in equilibrium. Given that the bank's expected profit function is concave in $\ell_i$ and that investors always recover their opportunity cost, the restriction on $c$ in Proposition 1 also guarantees that the equilibrium with $\ell^* = 2$ is the best achievable.

In either equilibrium asset structure, each bank has a portfolio of $1 + \ell^* = 3$ independently distributed projects with a distribution of returns as described in Table 1. For simplicity, we assume an equal probability of a project $i$ returning $R_H$ or $R_L$, that is $p = \frac{1}{2}$. This implies that all states are equally likely. Since there are 6 projects with two possible returns at date 2 each, there are $2^6 = 64$ states. Depending on the number of realizations of $R_L$ and $R_H$, there are 7 possible combinations of the 6 project returns numbered in the first column of the table. Each combination $(mR_L, (6-m)R_H)$, where $0 \leq m \leq 6$, is shown in the second column, and the number of states $\binom{6}{m}$ in which it occurs is in the third column. For example, there are $\binom{6}{3} = 20$ states where the combination of projects $(3R_L, 3R_H)$ occurs.

The next four columns in the table show bank $i$'s portfolio return $X_i$ for each combination of the 6 project returns. Given any $(mR_L, (6-m)R_H)$, bank $i$'s portfolio returns $X_i = \frac{kR_L+(3-k)R_H}{3}$, where $m \geq k$ and $0 \leq k \leq 3$, in $\binom{3}{k}\binom{3}{m-k}$ states. This is because for any given $(mR_L, (6-m)R_H)$ there are $\binom{3}{k}$ possible combinations of $kR_L$ and $(3-k)R_H$ in the 3 projects of bank $i$'s portfolio. For each of these combinations, the remaining $(m-k)R_L$ and $(3-(m-k))R_H$ returns can be combined in $\binom{3}{m-k}$ ways. For example, given the combination $(3R_L, 3R_H)$ of the 6 projects (that is, $m = 3$), $X_i = \frac{R_L+2R_H}{3}$ (that is, $k = 1$) realizes in $\binom{3}{1}\binom{3}{2} = 9$ states out of the 15 states with $3R_L$ and $3R_H$. Similarly for the remaining entries in the four columns. The final row gives the total of each column. For example, there are 24 out of the 64 states where $X_i = \frac{R_L+2R_H}{3}$ occurs.

As Table 1 shows, each bank $i$ has an identical portfolio distribution irrespective of the composition of the asset structure. What matters for the banks' portfolio returns with long-term financing is only the number of projects $\ell^*$ that each of them swaps in equilibrium, but not the resulting asset structure composition. This has direct implications for welfare. This is equal to the sum of a representative bank $i$'s expected profit and its investors' expected returns. Given that the investors always recover their opportunity cost, from (8) the equilibrium welfare per bank simplifies to

$$W(g) = E(X_i) - (1-\alpha)E(X_i < r) - 2c. \tag{9}$$

Given that each bank's portfolio return distribution is the same in either asset structure, all banks offer the same interest rate to investors and have the same bankruptcy probability in both structures. This gives the following result.

**Proposition 2** *With long-term finance, total welfare is the same in the clustered and unclustered structures.*

# 4 Short-term finance

We now analyze the case where banks use short-term finance and investors have per period opportunity cost $r_f$. As with long-term finance, we continue focusing on the clustered and unclustered structures with $\ell^* = 2$ and on the range $R_L < r_f^2 < \frac{5R_L+R_H}{6}$ so that bankruptcy occurs only when all projects in a bank's portfolio return $R_L$. We show that, in contrast to the case with long-term finance, the asset structure composition matters for systemic risk and total welfare when short-term finance is used. The reason is that the use of short-term debt may lead to information contagion among financial institutions. The extent to which this happens depends on the composition of the asset structure, that is on the degree of overlap of banks' portfolios.

The main difference with short-term finance is that it needs to be rolled over every period. If adverse interim information arrives, investors may not roll over the debt thus forcing the bank into early liquidation. We model this by assuming that a signal about future bank solvency arrives at date 1. The signal can either indicate the good news that all banks will be solvent at date 2 ($S = G$) or the bad news that at least one bank will default ($S = B$). The idea is that investors hear of an imminent bank failure and have to infer the prospects of their own bank. For simplicity, we assume that the signal does not reveal any information about any individual bank. This ensures that as far as individual investors are concerned, all banks look alike and have an equal probability of default once the signal arrives. We consider alternative information structures in Section 5.

Fig. 2 shows the sequence of events in the model with short-term finance. At date 0 each bank in the asset structure $g = \{C, U\}$ raises one unit of funds and promises investors an interest rate $r_{01}(g)$ at date 1. Investors know the asset structure, but do not know the position of any particular bank in the structure. At the beginning of date 1, before investors are repaid $r_{01}(g)$, the signal $S = \{G, B\}$ arrives. With probability $q(g)$ the signal $S = G$ reveals that all banks will be solvent at date 2. With probability $1 - q(g)$ the signal $S = B$ reveals that at least one bank will default at date 2. Upon observing the signal, investors decide whether to roll the funds over for a total promised repayment of $\rho_{12}^S(g)$ at date 2 or retain $r_{01}(g)$. If rollover occurs, the bank continues till date 2. Investors receive $\rho_{12}^S(g)$ and the bank $X_i - \rho_{12}^S(g)$ if it remains solvent. Otherwise, when the bank goes bankrupt, investors receive $\alpha X_i$ and the bank 0. If rollover does not occur, the bank is forced into early liquidation at date 1. Investors receive the proceeds from early liquidation, which for simplicity we assume to be equal to $r_f$, and the bank receives 0.

The interest rate $r_{01}(g)$ promised to investors at date 0 must be such that they recover their per period opportunity cost $r_f$ at date 1. Given that investors always recover their opportunity cost at date 1, irrespective of whether the bank is continued or liquidated at date 1, they will simply require a rate $r_{01}(g) = r_f$ at date 0.[4]

At date 1, after the signal $S$ is realized, investors roll over the debt if the promised repayment $\rho_{12}^S(g)$ is such that they can recover $r_{01}(g)r_f = r_f^2$ at date 2. When $S = G$ investors infer that they will always receive $\rho_{12}^G(g)$ at date 2 and thus roll over the debt for a repayment $\rho_{12}^G(g) = r_f^2$. When $S = B$, investors update the probability $\Pr(X_i \geq \rho_{12}^B(g)|B)$ that their bank will be able to repay them the promised repayment $\rho_{12}^B(g)$ at date 2. Then rollover occurs if there exists a value of $\rho_{12}^B(g)$ that satisfies investors' date 1 participation constraint

$$\Pr(X_i \geq \rho_{12}^B(g)|B)\rho_{12}^B(g) + \alpha E(X_i < \rho_{12}^B(g)|B) \geq r_f^2. \tag{10}$$

The first term is the expected return to investors conditional on $S = B$ when the bank remains solvent at date 2. The second term is their expected payoff conditional on $S = B$ when the bank defaults at date 2. This is equal to a fraction $\alpha$ of the bank's portfolio expected return $E(X_i < \rho_{12}^B(g)|B) = \sum_{x<\rho_{12}^B(g)} x \Pr(X_i = x|B)$. The equilibrium value of $\rho_{12}^B(g)$ if it exists, is the minimum promised repayment that satisfies (10) with equality and minimizes the probability of bank default conditional on $S = B$.

The expected profit of bank $i$ at date 0 depends on the realization of the signal and on the investors' rollover decision at date 1. When rollover occurs and the bank continues at date 1, its expected profit is simply given by

$$\pi_i(g) = E(X_i) - r_f^2 - (1 - q(g))(1 - \alpha)E(X_i < \rho_{12}^B(g)|B) - 2c. \tag{11}$$

As with long-term debt, the bank's expected profit in the case of rollover can be expressed by the expected return of its portfolio $E(X_i)$ minus the investors' opportunity cost $r_f^2$, the expected bankruptcy costs $(1-q(g))(1-\alpha)E(X_i < \rho_{12}^B(g)|B)$, and the total due diligence

[4] If investors obtained only $\beta r_f$ with $\beta < 1$ as early liquidation proceeds, they would require $r_{01}(g) > r_f$ when they anticipate not rolling over the debt at date 1. This would imply higher deadweight costs and lower welfare with early liquidation, but our qualitative results would be similar.

costs $2c$.

When, after the realization of a bad signal, rollover does not occur, the bank is early liquidated at date 1 and receives 0. Then, its expected profit, given by

$$\pi_i(g) = q(g)\left[E(X_i \geq r_f^2|G) - r_f^2\right] - 2c, \tag{12}$$

is positive only when with probability $q(g)$ the good signal arrives. Note that (11) and (12) imply that, in a given asset structure $g$, the bank has higher expected profit when debt is rolled over at date 1 than when it is not.

## *4.1 Investors' rollover decisions at date* 1

The crucial difference between long-term and short-term finance is that in the latter case the asset structure matters for the equilibrium interest rates, bank profits and ultimately total welfare. The reason is that the probability distribution of the signal and the associated conditional probabilities of bank default at date 2 differ in the two structures. To see this, consider first the distribution of the signal. The good signal arrives when all banks' portfolios return at least $(2R_L + R_H)/3$ and investors can obtain the opportunity cost $r_f^2$ at date 2. Thus, the probability of $S = G$ is

$$q(g) = \Pr(\bigcap_{i=1}^{6} X_i \geq r_f^2), \tag{13}$$

where $\Pr(\bigcap_i (X_i \geq r_f^2) = \Pr(X_1 \geq r_f^2, X_2 \geq r_f^2, ..., X_6 \geq r_f^2)$ represents the probability that none of the six banks defaults. By contrast, the bad signal arrives when the portfolio of at least one bank returns $X_i = R_L < r_f^2$. Thus, the probability of $S = B$ is

$$1 - q(g) = \Pr(\bigcup_{i=1}^{6} X_i < r_f^2) = \Pr(\bigcup_{i=1}^{6} X_i = R_L), \tag{14}$$

where $\Pr(\bigcup_{i=1}^{6} X_i = R_L)$ is the probability that at least one of the six banks defaults.

The clustered and unclustered asset structures entail different composition of banks' portfolios. In the former banks hold identical portfolios within each cluster. In the latter each bank shares projects with two others but no banks hold identical portfolios. This implies a different concentration of defaults in the two asset structures. In the clustered structure defaults occur in groups. The 3 banks in one cluster default when all the 3 projects in their portfolios return $R_L$ or all 6 banks default when all the 6 projects in the economy give $R_L$. In the unclustered structure defaults are more scattered. As banks hold diverse portfolios, each bank can fail independently of the others. When the 3 projects in one bank's portfolio return $R_L$, only that bank defaults. As the number of projects returning $R_L$ increases, more banks also default in the unclustered structure. The different concentration of defaults implies different probability distributions of the signal in the two asset structures. Formally, the probability of $S = B$ is given by

$$1 - q(C) = 2\sum_{m=3}^{6} \frac{\binom{6-3}{6-m}}{2^6} - \frac{1}{2^6} = \frac{15}{64}, \tag{15}$$

in the clustered structure, and by

$$1 - q(U) = 6\sum_{m=3}^{6} \frac{\binom{6-3}{6-m}}{2^6} - 6\sum_{m=4}^{6} \frac{\binom{6-4}{6-m}}{2^6} + \frac{1}{2^6} = \frac{25}{64} \tag{16}$$

in the unclustered structure, where as before $m$ is the number of projects returning $R_L$ for a given combination $(mR_L, (6-m)R_H)$ of the 6 projects in the economy.[5] The bad signal arrives when at least three projects forming a bank's portfolio return $X_i = R_L$. In the clustered structure this occurs in $2\binom{6-3}{6-m}$ out of the $2^6 = 64$ states for any given combination $(mR_L, (6-m)R_H)$ of projects with $m \geq 3$. Summing up the combinations with $m \geq 3$ and taking into account that there is only one state where $m = 6$ gives (15). Similar considerations explain (16). The higher number of default states in the unclustered structure (25 against 15) follows directly from the higher concentration of defaults when banks are clustered.

[5] See Appendix B of Allen, Babus and Carletti (2011) for a full derivation of (15) and (16).

It follows that the probability of $S = G$ is

$$q(C) = \frac{49}{64} \text{ and } q(U) = \frac{39}{64} \tag{17}$$

in the clustered and unclustered asset structures, respectively, so that clearly

$$q(C) > q(U). \tag{18}$$

What matter for investors' rollover decisions are the conditional probability distributions of banks' portfolio returns. Tables 2 and 3 show these for the clustered and unclustered asset structures, respectively. Both tables report the conditional distributions for each combination $(mR_L, (6-m)R_H)$ of project realizations and in total. The first two columns in the tables number and describe the combinations $(mR_L, (6-m)R_H)$. The third column shows the number of states where the bad signal arrives at date 1 and at least one bank will default at date 2. The fourth set of columns shows bank $i$'s portfolio distribution conditional on $S = B$. The next two sets of columns show the number of no default states and bank $i$'s portfolio distribution conditional on $S = G$. Note that the distribution of $X_i$ conditional on $S = G$ is simply the difference between the unconditional probability distribution of $X_i$ as described in Table 1 and the conditional distribution on $S = B$, that is $\Pr(X_i = x|G) = \Pr(X_i = x) - \Pr(X_i = x|B)$. Finally, the last row in both tables shows the total number of states where the bad and good signals arrive out of the 64 states and the total number of states for the conditional distributions of $X_i$.[6]

Comparing Tables 2 and 3, it can be seen that the conditional distributions of banks' portfolio returns are quite different in the two asset structures. In particular, the probability of $X_i = R_L$ conditional on $S = B$ in the clustered structure, which is equal to $\frac{8}{15}$, is much higher than in the unclustered structure, where it is $\frac{8}{25}$. This also implies that the conditional probability $\Pr(X_i \geq \rho_{12}^B(g)|B)$ that the bank is solvent and repays $\rho_{12}^B(g)$

[6] See Appendix C of Allen, Babus and Carletti (2011) for a full explanation of the probability distributions in Tables 2 and 3.

to the investors at date 2 conditional on $S = B$ is higher in the unclustered than in the clustered structure. That is,

$$\Pr(X_i \geq \rho_{12}^B(U)|B) > \Pr(X_i \geq \rho_{12}^B(C)|B) \tag{19}$$

for $\rho_{12}^B(g) \in [R_L, \frac{2R_L+R_H}{3}]$. This difference means that investors' rollover decisions can differ between the two asset structures. We study the clustered structure first.

**Proposition 3** *With short-term finance, when the bad signal ($S = B$) is realized in the clustered structure and $R_H > \frac{13}{12}R_L$, there exists $\alpha_{MID}(C) < \alpha_{LOW}(C)$ such that*

*(i). For $\alpha \geq \alpha_{LOW}(C)$, investors roll over the debt for a promised repayment $\rho_{12}^B(C) \in [r_f^2, \frac{2R_L+R_H}{3}]$.*

*(ii). For $\alpha_{MID}(C) \leq \alpha < \alpha_{LOW}(C)$, investors roll over the debt for a promised repayment $\rho_{12}^B(C) \in [\frac{2R_L+R_H}{3}, \frac{R_L+2R_H}{3}]$.*

*(iii+iv). For $\alpha < \alpha_{MID}(C)$, investors do not roll over the debt and the bank is liquidated early at date 1.*

**Proof.** See the Appendix, where the expressions $\alpha_{MID}(C)$ and $\alpha_{LOW}(C)$ are also provided. ■

The proposition is illustrated in Fig. 3, which plots investors' rollover decisions as a function of the exogenous parameters $\alpha$ and $r_f^2$. The result follows immediately from the investors' participation constraint at date 1. When the bad signal is realized, the bank continues at date 1 whenever investors can be promised a repayment that satisfies (10). Whether this is possible depends on the fraction $\alpha$ of the bank's portfolio return accruing to the investors when the bank defaults at date 2 and on their opportunity cost $r_f^2$ over the two periods. When $\alpha$ is high or $r_f^2$ is low as in Region i in Fig. 3, there exists a repayment $\rho_{12}^B(C)$ that satisfies (10). Investors roll over the debt and the bank continues. The promised repayment compensates the investors for the possibility that they obtain only $\alpha X_i$ in the case of default. Given $\alpha$ is high, $\rho_{12}^B(C)$ does not need to be high for

(10) to be satisfied. Thus, the equilibrium $\rho_{12}^B(C)$ lies in the lowest interval of the bank's portfolio return, $[r_f^2, \frac{2R_L+R_H}{3}]$.

As $\alpha$ decreases or $r_f^2$ increases so that Region ii is reached, investors still roll over the debt but require a higher promised repayment as compensation for the greater losses in the case of bank default. Thus, $\rho_{12}^B(C)$ is higher and lies in the interval $[\frac{2R_L+R_H}{3}, \frac{R_L+2R_H}{3}]$. This also implies that, conditional on the realization of the bad signal, bankruptcy occurs at date 2 not only when a bank's portfolio pays off $X_i = R_L$ but also when it pays $X_i = \frac{2R_L+R_H}{3}$.

As $\alpha$ decreases or $r_f^2$ increases further so that Regions iii and iv below $\alpha_{MID}(C)$ are reached, it is no longer possible to satisfy (10) for any $\rho_{12}^B(g) \leq R_H$. Then, investors do not roll over the debt and the bank is liquidated early at date 1.

A similar result holds for the unclustered structure.

**Proposition 4** *With short-term finance, when the bad signal ($S = B$) is realized in the unclustered structure, there exists $\alpha_{LOW}(U)$ such that*

*(i+ii+iii). For $\alpha \geq \alpha_{LOW}(U)$, investors roll over the debt for a promised repayment $\rho_{12}^B(U) \in [r_f^2, \frac{2R_L+R_H}{3}]$.*

*(iv). For $\alpha < \alpha_{LOW}(U)$, investors do not roll over the debt and the bank is liquidated at date 1.*

**Proof.** See the Appendix, where the expression for $\alpha_{LOW}(U)$ is also provided. ■

Proposition 4 is also illustrated in Fig. 3. As in the clustered structure, investors roll over the debt when there exists a repayment $\rho_{12}^B$ that satisfies their participation constraint (10) with equality. Whether such a repayment exists depends as before on the parameters $\alpha$ and $r_f^2$. When they lie in the Regions i, ii and iii above $\alpha_{LOW}(U)$, the probability $\Pr(X_i \geq \rho_{12}^B(U)|B)$ is sufficiently high to ensure that (10) is always satisfied for a repayment $\rho_{12}^B(U)$ in the interval $[r_f^2, \frac{2R_L+R_H}{3}]$. However, when $\alpha$ and $r_f^2$ lie in Region iv (10) can no longer be satisfied and the bank is liquidated early.

### 4.2 Welfare with short-term finance

We next consider welfare in the two asset structures with short-term finance. As with long-term finance, in both structures we can focus on the total welfare per bank as defined by the sum of a representative bank $i$'s expected profit and its investors' expected returns. Welfare now depends on the investors' rollover decisions, since these affect the bank's expected profit. Using (11) and (12), welfare is given by

$$W(g) = E(X_i) - (1 - q(g))(1 - \alpha)E(X_i < \rho_{12}^B(g)|B) - 2c \tag{20}$$

when the bank is continued till date 2 and by

$$W(g) = q(g)\left[E(X_i \geq r_f^2|G)\right] + (1 - q(g))r_f^2 - 2c \tag{21}$$

when the bank is liquidated at date 1 after the arrival of the bad signal. In (20) welfare equals the expected return of bank portfolio $E(X_i)$ minus the expected bankruptcy costs $(1 - q(g))(1 - \alpha)E(X_i < \rho_{12}^B(g)|B)$ and the due diligence costs $2c$. In contrast, in (21) welfare is given by the sum of the expected return of the bank portfolio conditional on $S = G$, $q(g)\left[E(X_i \geq r_f^2|G)\right]$, and the date 2 value of the liquidation proceeds $(1 - q(g))r_f^2$ minus the due diligence costs $2c$.

Using (20) and (21) it is easy to derive the expressions for the welfare in the two asset structures. The following holds.

**Proposition 5** *The comparison of total welfare in the two structures is as follows: There exists $\alpha_W < \alpha_{LOW}(C)$ such that*

*(i). For $\alpha \geq \alpha_{LOW}(C)$, total welfare is the same in the clustered and unclustered structures.*

*(ii+iii1). For $\alpha_W < \alpha < \alpha_{LOW}(C)$, total welfare is higher in the unclustered structure than in the clustered structure.*

*(iii2+iv). For $\alpha < \alpha_W$, total welfare is higher in the clustered structure than in the*

*unclustered structure.*

**Proof.** See the Appendix, where the expression for $\alpha_W$ is also provided. ■

Fig. 4 illustrates the proposition by showing the welfare in the clustered and unclustered structures. The crucial point is that with short-term finance total welfare depends on the asset structure. Which is better depends crucially on the parameters $\alpha$ and $r_f^2$. As (20) shows, $\alpha$ affects welfare when investors roll over as it determines the size of the expected bankruptcy costs in the case of bank default. As (21) shows, $r_f^2$ affects welfare when the bank is liquidated early as a measure of the liquidation proceeds.

In Region i, where $\alpha \geq \alpha_{LOW}(C)$, investors roll over the debt for a promised total repayment $\rho_{12}^B(C) \in [r_f^2, \frac{2R_L+R_H}{3}]$ in both asset structures. In either of them, banks default when their portfolios pay off $R_L$ and make positive profits in all the other states. As with long-term finance, total welfare is then the same in both asset structures.

In Region ii, where $\alpha$ lies in between $\alpha_{MID}(C)$ and $\alpha_{LOW}(C)$, rollover occurs in both asset structures, but investors require a higher promised repayment in the interval $[\frac{2R_L+R_H}{3}, \frac{R_L+2R_H}{3}]$ in the clustered structure. This implies higher expected bankruptcy costs and thus lower welfare in the clustered structure as banks also default when their portfolios return $X_i = \frac{2R_L+R_H}{3}$.

In Regions iii1 and iii2 rollover occurs in the unclustered structure but not in the clustered one. Total welfare is then given by (20) and (21) in the unclustered and clustered structures, respectively. In the former, welfare is decreasing in the bankruptcy costs, $1-\alpha$. In the latter, welfare is increasing with $r_f^2$ as it increases the liquidation proceeds. As $\alpha$ falls and $r_f^2$ increases, total welfare in the unclustered structure becomes equal to that in the clustered structure, and it then drops below.

Finally, in Region iv, where $\alpha \leq \alpha_{LOW}(U)$, banks are always liquidated early after the arrival of the bad signal so that welfare is given by (21) in both asset structures. Since, as (18) shows, the good signal occurs more often in the clustered structure, the expected return $q(g)\left[E(X_i \geq r_f^2|G)\right]$ is higher in the clustered structure while the date 2 value of the early liquidation proceeds $(1-q(g))r_f^2$ is higher in the unclustered structure. The first term dominates so that total welfare is greater in the clustered structure.

To sum up, in contrast to the case with long-term finance, the composition of the asset structure matters for investors' rollover decisions and thus total welfare with short-term finance. Comparing Propositions 3 and 4 shows that rollover occurs for a larger parameter space in the unclustered structure than in the clustered one. This implies that there is more systemic risk in concentrated than in dispersed asset structures. However, the latter do not always entail higher welfare. The reason is that, as defaults are less concentrated, the bad signal arrives more often in dispersed structures. Whether this also leads to lower welfare depends on the size of the bankruptcy costs and on the proceeds from early liquidation.

The basic analysis we have done so far has the following features. First, the signal that investors receive at the interim date with short-term debt is imperfect. Since banks are opaque, the signal reveals only information about a bank's overall solvency state rather than about the precise value of its portfolio. Second, the analysis has so far concentrated on the implications of different debt maturities and asset structures on rollover risk and total welfare, without looking at banks' choice of optimal debt maturity. Finally, the model has shown that multiple asset structures are possible in equilibrium because banks cannot coordinate on the composition of the asset structure when exchanging projects. If this was possible, only the efficient structure would emerge. We next relax these assumptions.

# 5 Extensions

In this section we discuss different types of signal arriving at the interim date, banks' choice of long-term versus short-term finance, and different types of coordination mechanisms in the formation of asset structures.

### *5.1 Information structure*

The core of our analysis is the interaction between the interim information arriving at date 1, the composition of banks' asset structure, and the funding maturity. Interim information has been modeled as a signal indicating whether at least one bank will default at date 2. The idea is that banks' assets are opaque, particularly in periods of crises (see, e.g., Morgan, 2004; Flannery, Kwan, and Nimalendran, 2010). This implies that observed signals in the markets do not typically reveal the precise value of banks' asset fundamentals but rather disclose information on the overall outcome of a bank's assets relative to its liabilities. For simplicity, we also suppose that the signal does not reveal the identity of potentially failing banks and all investors and banks are treated alike. Investors know the asset structure but do not know any bank's position in it. Upon observing the signal, they update the conditional probability that their own bank will default at date 2. The crucial feature for our result is that the signal generates a different information partition of the states and thus different conditional probabilities of default in the two asset structures. This implies different rollover decisions and thus different welfare in the two structures with short-term finance.

Any signal that generates different information partitions and leads to different conditional probabilities across asset structures will have the same qualitative effect as in our basic model. Examples are signals indicating that a particular bank has gone bankrupt or that a particular real sector is more likely to fail. Both of these signals would indicate in our model that a particular project or set of projects has a higher default probability than originally believed. This would generate different information partitions on banks' future defaults depending on the different compositions of banks' asset structures and would thus lead to different conditional probabilities in the two structures.

An alternative (but less plausible given banks' asset opacity) signal that would not lead to differences in the two asset structures is one carrying generic information about the underlying fundamentals. An example is a signal indicating the number of projects returning $R_L$ in the economy (without specifying the identity of these projects). This would simply reveal which state of the economy or combination $(mR_L, (6-m)R_H)$ of projects has been realized and the consequent conditional distribution of returns. As Table 1 shows, the conditional distribution would be the same in the two asset structures, as with long-term debt. This would lead to the same investor rollover decisions and welfare in the two structures. This means that in our model bank level information about defaults or specific information on defaulting sectors is different from generic information about fundamentals. The former interacts with the composition of the asset structure in generating systemic risk, while the latter does not.

The result that information about defaults is very different from information about project outcomes holds beyond our basic model. Given any number of banks above six and of connections, the probability distribution conditional on an interim signal revealing the number of low and high return projects will be independent of the composition of banks' asset structure. The possible combinations of project outcomes will be the same for a given number of connections irrespective of the architecture of the asset structure.

### *5.2 Long-term versus short-term finance*

So far we have considered long-term and short-term finance separately and we have shown that the latter entails rollover risk while the former does not. This raises the question as to why banks use short-term finance in the first place. There are a number of theories justifying its use. Flannery (1986) and Diamond (1991) suggest that short-term finance can help overcome asymmetric information problems in credit markets. Calomiris and Kahn (1991) and Diamond and Rajan (2001) argue that short-term debt can play a role as a discipline device to ensure that managers behave optimally. Brunnermeier and Oehmke (2009) suggest that creditors shorten the maturity of their claims to obtain priority, leading to an excessive use of short-term debt. Another important rationale for the use of short-term debt is an upward sloping yield curve. Borrowing short-term at low rates to finance high yielding long-term assets allows significant profits to be made and this is the approach used here.

In our model the choice of the optimal maturity structure depends on the difference between the long-term rate $r_F^2$ and the short-term rate $r_f^2$. To see this, suppose that once the asset structure is determined, banks choose the maturity of the debt that maximizes their expected profits. With short-term debt bank expected profit is given by (11) and (12) depending on the investors' rollover decisions as described in Propositions 3 and 4. With long-term debt bank expected profit is always given by (8). Comparing the different expressions for bank expected profits with short-term and long-term financing profits gives the following.

**Proposition 6** *Let $\overline{r}_F^2(C)$ and $\overline{r}_F^2(U)$ be the set of indifference points for which bank expected profit is the same with short-term and long-term debt in the clustered and unclustered structures, respectively. Then the optimal debt maturity structure is as follows:*

1. *For $r_F^2 \geq \max\left\{\overline{r}_F^2(C), \overline{r}_F^2(U)\right\}$ short-term debt is optimal in both structures.*
2. *For $\overline{r}_F^2(C) > r_F^2 \geq \overline{r}_F^2(U)$ short-term debt is optimal in the unclustered structure and long-term debt is optimal in the clustered structure.*
3. *For $\overline{r}_F^2(U) > r_F^2 \geq \overline{r}_F^2(C)$ short-term debt is optimal in the clustered structure and long-term debt is optimal in the unclustered structure.*
4. *For $r_F^2 < \min\left\{\overline{r}_F^2(C), \overline{r}_F^2(U)\right\}$ long-term debt is optimal in both structures.*

**Proof.** See the Appendix, where the expressions for the boundaries $\overline{r}_F^2(C)$ and $\overline{r}_F^2(U)$ are also provided. ■

The proposition is illustrated in Fig. 5, which plots the bank's choice of debt maturity structure as a function of the rates $r_F^2$ and $r_f^2$ for a given value of the fraction $\alpha$ of the bank's portfolio return that investors receive in case of default. The boundaries $\overline{r}_F^2(C)$ and $\overline{r}_F^2(U)$ represent the combinations of $r_f^2$ and $r_F^2$ such that bank expected profit is the same with short-term and long-term debt in the clustered structure and in the unclustered structure, respectively. Both $\overline{r}_F^2(C)$ and $\overline{r}_F^2(U)$ are piecewise linear functions of $r_f^2$ since bank expected profit with short-term debt changes with investors' rollover decisions. Consider, for example, $\overline{r}_F^2(C)$. For values of $r_f^2$ in Region i of Proposition 3, rollover occurs for a repayment $\rho_{12}^B(g) \in [r_f^2, \frac{2R_L+R_H}{3}]$ and the bank expected profit $\pi_i(C)$ is given by (11). When $r_f^2$ is in Region ii, the repayment increases to $\rho_{12}^B(g) \in [\frac{2R_L+R_H}{3}, \frac{R_L+2R_H}{3}]$. This lowers $\pi_i(C)$ and $\overline{r}_F^2(C)$ jumps up. As $r_f^2$ enters Region iii+iv the bank is liquidated early and $\pi_i(C)$ is given by (12). Thus, $\overline{r}_F^2(C)$ jumps even higher. Similar considerations hold for $\overline{r}_F^2(U)$, where the expression for this depends on which region of Proposition 4 $r_f^2$ lies in.

Proposition 6 has important implications. First, it shows that, for a given asset structure, the optimality of short-term debt declines with the rate $r_f^2$. The reason is that an increase in $r_f^2$ across the different regions leads investors to either require a higher repayment $\rho_{12}^B$ or force the bank into liquidation. Both of these reduce the bank expected profit in a given asset structure, and thus the optimality of short-term debt relative to long-term debt. Second, Proposition 6 shows that the optimality of the short-term debt depends on the asset structure. Short-term debt is optimal in both structures in Region 1, but is optimal only in one structure in Regions 2 and 3. The bank's choice of the optimal debt maturity conditional on the asset structure is always efficient from a welfare perspective. The reason is that, as investors always obtain their opportunity cost, total welfare coincides with bank expected profits. Thus, the choice of the optimal debt maturity resembles the comparison of total welfare in the two asset structures as described in Proposition 5. When welfare is higher in the unclustered structure (as in Regions ii+iii1 of Proposition 5), the bank's expected profit will also be higher. This corresponds to Region 2 of Proposition 6 where short-term debt is only optimal in the unclustered structure. Similar considerations hold for the other regions. Finally, note that short-term debt is never optimal only in the unlikely case of Region 4 where the long-term rate $r_f^2$ is smaller than the short-term one $r_F^2$.

### 5.3 *What is the market failure?*

An important feature of the network literature and of the equilibrium concept of Jackson and Wolinsky (1996) that we use is that banks are not able to determine the composition of the asset structure. Each bank individually chooses the number of links it wishes to have taking as given the choices of the other banks. Since banks form links simultaneously, with $\ell^* = 2$ either a clustered or an unclustered structure can emerge. With long-term finance the multiplicity of asset structures does not matter. However, with short-term finance it does matter since systemic risk and welfare differ in the two structures. Investors are indifferent as they obtain their opportunity cost in either asset structure, but banks would clearly prefer the structure that gives them higher expected profits. The market failure in our model lies precisely in banks' inability to choose the composition of the asset structure explicitly.

The choice of the optimal debt maturity structure can be seen as a constrained efficient solution, given the multiplicity of the asset structures. By choosing the maturity of their debt, banks can optimize their expected profits and welfare conditional on the asset structure. Ideally, any mechanism that would instead allow banks to coordinate and choose the preferred structure would achieve efficiency. One example of such a mechanism is to have banks enforce exclusive contracts that condition their linkages on the connections between all other banks in the system. This arrangement would make it possible to ensure that only efficient structures are implemented. However, as banks cannot observe the linkages of the other banks, exclusive contracting in linkage formation is not possible in our model (see, e.g., Bisin and Gottardi, 2006; Bizer and De Marzo, 1992).

Similarly, government regulation, centralized exchanges or clearinghouses could potentially be used to ensure that only the efficient asset structure is chosen. Clearing bank linkages through a centralized exchange or a clearinghouse rather than through bilateral transactions would improve transparency concerning the composition of banks' asset structure. The exchange or the clearinghouse could implement the efficient asset structure by allocating counterparts to correspond to the clustered or unclustered structure.

However, all these solutions seem hard to implement in practice. Private coordination among banks would be difficult to achieve and sustain, particularly as the number of banks grows large. Government intervention would require the gathering of a significant amount of information from banks about their linkages and the determination and implementation of the optimal asset structure. Similarly for mechanisms operating through centralized exchanges or clearinghouses. Exchanges ensure the existence of a matching mechanism to pair traders' orders rather than choosing a particular asset structure. Clearinghouses clear trades to reduce settlement risks. Transforming them into mechanisms that implement the optimal asset structure is challenging.

One possibility is that a clearinghouse guarantees asset swap payoffs based on an ex-ante capital contribution by banks that would be greater the higher the systemic risk. Banks would then have an incentive to converge towards the structure with lower systemic risk. This scheme implements the welfare optimal asset structure for most of the parameter space. However, in Region iii2 in Fig. 4, the unclustered structure, which does not entail systemic risk, is inferior and so this scheme would not be desirable. In order to be always able to implement the optimal welfare asset structure, the clearinghouse would need to be able to distinguish between the regions and make the ex-ante capital charge increase or decrease with systemic risk as appropriate. Again, this would require the clearinghouse to have a large amount of information. These issues are an important topic for future research.

## 6 Concluding remarks

Understanding asset commonality among financial institutions is important for understanding systemic risk. In this paper we have developed a model where asset commonality arises from asset swaps, and we have shown that the composition of banks' asset structures interacts with the funding maturity in determining systemic risk.

The asset structure matters for systemic risk and total welfare when banks use short-term finance, but not when they use long-term finance. The reason is that with short-term finance banks are informationally linked. When adverse interim information on banks' future solvency arrives, investors update the default probability of their own bank and decide whether to roll over the debt. This inference problem depends on the structure of bank assets. In concentrated structures defaults are more correlated than in dispersed structures. This means that a negative interim signal conveys worse information and rollover occurs less often in the former than in the latter structure. In other words, there is more systemic risk in concentrated than in dispersed asset structures.

The key trade off between the clustered and the unclustered structures in our framework derives from the different overlap and risk concentration among banks' portfolios in the two asset structures. While we have analyzed an economy where each of the six banks swap two projects, the results hold more generally. What matters is that the multiple asset structures that emerge in equilibrium differ in terms of banks' asset concentration. An increase in the number of banks in the economy would increase the multiplicity of equilibrium asset structures. Still there would exist clustered structures where banks have highly correlated portfolios and dispersed structures where banks have more diverse portfolios, as in Fig. 1. As in our basic model, investors' rollover decisions and thus welfare would then still differ in the two types of asset structures.

We model asset commonality through asset swaps. This allows us to use a standard approach based on network formation and to focus on multiple asset structures. However, the insights of our model hold more generally. Any mechanism leading to similar asset structures would lead to analogous results. An example is banks' lending choices. A concentrated asset structure would arise if groups of banks lend to different sectors; for instance some banks do retail mortgage lending and others do commercial mortgage lending. A dispersed, or unconcentrated, asset structure would instead arise if all banks lend to the same sectors but in different shares or in different geographical areas. In this case all banks have some assets in common but maintain distinct portfolios.

# Appendix

**Proof of Proposition 1**. Given the focus on the situation that bankruptcy only occurs when all projects in a bank's portfolio return $R_L$, a bank's expected profit (6) with $\ell = 2$ simplifies tosimplifies to

$$\pi_i(g) = E(X_i) - r_F^2 - (1-p)^3(1-\alpha)R_L - 2c.$$

To show pairwise stability, we first consider severing a link. Suppose that bank 1 severs the link with bank 3 so that its portfolio is now $\frac{2}{3}\theta_1 + \frac{1}{3}\theta_2$ and its profit is

$$\pi_1(g - \ell_{13}) = E(X_i) - r_F^2 - (1-p)^2(1-\alpha)R_L - c.$$

Bank 1 does not deviate if $\pi_i(g) \geq \pi_1(g - \ell_{13})$, which is satisfied for $c \leq p(1-p)^2 R_L$.

Suppose now that bank 1 adds a link with bank 4 so that its portfolio is now $\frac{1}{6}\theta_1 + \frac{1}{3}\theta_2 + \frac{1}{3}\theta_3 + \frac{1}{6}\theta_4$ and its profit is

$$\pi_1(g + \ell_{14}) = E(X_i) - r_F^2 - (1-p)^4(1-\alpha)R_L - 3c$$

when bankruptcy occurs when all projects pay off $R_L$. If bankruptcy occurs more often than this, the expected profit from the deviation will be lower. Thus, it is sufficient for the deviation not to be profitable that $\pi_i(g) \geq \pi_1(g+\ell_{14})$ which requires $c \geq p(1-p)^3(1-\alpha)R_L$. Since all banks are symmetric, this shows that $\ell^* = 2$ is a pairwise stable equilibrium for the range of $c$ given in the proposition.

To see that $\ell^* = 2$ is the Pareto dominant equilibrium it is sufficient to show that the bank's expected profit is highest in this case since the investors always obtain their opportunity cost. First note that (8) is concave in $\ell$. Combining this with the condition that $c$ lies in the range given in the proposition, it follows that a bank's expected profit in the equilibrium with $\ell^* = 2$ is greater than in either the equilibrium with $\ell^* = 1$ or $\ell^* = 3$ or any other equilibrium. $\square$

**Proof of Proposition 3**. We proceed in two steps. First, we find the minimum value of $\alpha$ as a function of the short-term risk free rate $r_f^2$ in each interval of the bank's portfolio return $X_i$ such that investors' participation constraint (10) is satisfied for a feasible promised repayment $\rho_{12}^B(C)$. Second, we compare the functions representing the minimum values of $\alpha$ found in the first step to find the equilibrium value of $\rho_{12}^B(g)$.

*Step 1.* We start by determining the minimum value of $\alpha$ such that (10) is satisfied for $\rho_{12}^B(C) \in [r_f^2, \frac{2R_L+R_H}{3}]$. Substituting $\rho_{12}^B(C) = \frac{2R_L+R_H}{3}$ in (10) and using the distribution probability $\Pr(X_i = x|B)$ as in Table 3, we obtain

$$\frac{7}{15}\frac{2R_L + R_H}{3} + \alpha\frac{8}{15}R_L = r_f^2,$$

from which

$$\alpha_{LOW}(C) = \frac{45r_f^2 - 7(2R_L + R_H)}{24R_L}.$$

This implies that for any $\alpha \geq \alpha_{LOW}(C)$, there exists a value of $\rho_{12}^B(C) \in [r_f^2, \frac{2R_L+R_H}{3}]$ such that investors roll over their debt. Analogously, for $\rho_{12}^B(C) \in [\frac{2R_L+R_H}{3}, \frac{R_L+2R_H}{3}]$, we obtain

$$\frac{4}{15}\frac{R_L + 2R_H}{3} + \alpha(\frac{8}{15}R_L + \frac{3}{15}\frac{2R_L + R_H}{3}) = r_f^2$$

from which

$$\alpha_{MID}(C) = \frac{45r_f^2 - 4R_L - 8R_H}{3(10R_L + R_H)}.$$

Finally, for $\rho_{12}^B(C) \in [\frac{R_L+2R_H}{3}, R_H]$ we obtain

$$\frac{1}{15}R_H + \alpha(\frac{8}{15}R_L + \frac{3}{15}\frac{2R_L + R_H}{3} + \frac{3}{15}\frac{R_L + 2R_H}{3}) = r_f^2$$

from which

$$\alpha_{HIGH}(C) = \frac{15r_f^2 - R_H}{11R_L + 3R_H}.$$

The interpretation of $\alpha_{MID}(C)$ and $\alpha_{HIGH}(C)$ is the same as the one for $\alpha_{LOW}(C)$.

*Step 2.* To find the equilibrium value of $\rho_{12}^B(C)$ defined as the minimum promised repayment that satisfies (10), we now compare the functions $\alpha_{LOW}(C)$, $\alpha_{MID}(C)$ and $\alpha_{HIGH}(C)$. We then obtain:

$$\alpha_{MID}(C) - \alpha_{LOW}(C) = \frac{7R_H^2 + 20R_H R_L + 108R_L^2 - 45r_f^2(2R_L + R_H)}{24R_L(10R_L + R_H)}.$$

We note that $\alpha_{MID}(C) - \alpha_{LOW}(C)$ is positive for $r_f^2 < \overline{r}_f^2 = \frac{7R_H^2+20R_H R_L+108R_L^2}{45(2R_L+R_H)} < \frac{5R_L+R_H}{6}$, and negative otherwise. Similarly, it can be shown that $\alpha_{HIGH}(C) - \alpha_{MID}(C) > 0$ for any $r_f^2 \in [\overline{r}_f^2, \frac{5R_L+R_H}{6}]$ and $R_H > \frac{13}{12}R_L$, while $\alpha_{HIGH}(C) - \alpha_{LOW}(C) > 0$ for any $r_f^2 \in [R_L, \overline{r}_f^2]$. Given that in equilibrium the bank offers the minimum level of $\rho_{12}^B(C)$ that satisfies (10), the proposition follows. $\square$

**Proof of Proposition 4**. We proceed in two steps as in the proof of Proposition 3.

*Step 1.* We determine first the minimum value of $\alpha$ such that (10) is satisfied for $\rho_{12}^B(U) \in [r_f^2, \frac{2R_L+R_H}{3}]$. Substituting $\rho_{12}^B(U) = \frac{2R_L+R_H}{3}$ in (10) and using the distribution probability $\Pr(X_i = x|B)$ as in Table 4, we obtain

$$\frac{17}{25}\frac{2R_L + R_H}{3} + \alpha\frac{8}{25}R_L = r_f^2,$$

from which

$$\alpha_{LOW}(U) = \frac{75r_f^2 - 17(2R_L + R_H)}{24R_L}.$$

As before, this implies that for any $\alpha \geq \alpha_{LOW}(U)$, there exists a value of $\rho_{12}^B(U) \in [r_f^2, \frac{2R_L+R_H}{3}]$ such that investors roll over their debt. Analogously, for $\rho_{12}^B(U) \in [\frac{2R_L+R_H}{3}, \frac{R_L+2R_H}{3}]$ and $\rho_{12}^B(U) \in [\frac{R_L+2R_H}{3}, R_H]$, respectively, we obtain

$$\frac{6}{25}\frac{R_L+2R_H}{3}+\alpha(\frac{8}{25}R_L+\frac{11}{25}\frac{2R_L+R_H}{3})=r_f^2$$

from which

$$\alpha_{MID}(U)=\frac{75r_f^2-6(R_L+2R_H)}{46R_L+11R_H};$$

and

$$\frac{1}{25}R_H+\alpha(\frac{8}{25}R_L+\frac{11}{25}\frac{2R_L+R_H}{3}+\frac{5}{25}\frac{R_L+2R_H}{3})=r_f^2$$

from which

$$\alpha_{HIGH}(U)=\frac{25r_f^2-R_H}{17R_L+7R_H}.$$

*Step 2.* We now compare the functions $\alpha_{LOW}(U)$, $\alpha_{MID}(U)$ and $\alpha_{HIGH}(U)$ to find the equilibrium value of $\rho_{12}^B(C)$. After some algebraic manipulation it is possible to see that $\alpha_{LOW}(U)<\alpha_{MID}(U)<\alpha_{HIGH}(U)$ for any $r_f^2\in[R_L,\frac{5R_L+R_H}{6}]$. Thus, the proposition follows given that the bank always offers investors the minimum total repayment that satisfies (10). $\square$

**Proof of Proposition 5**. The proposition follows immediately from the comparison of total welfare in the two asset structures in the different regions. We analyze each region in turn.

*Region i.* For $\alpha\geq\alpha_{LOW}(C)>\alpha_{LOW}(U)$, (10) is satisfied for $\rho_{12}^B(g)\in[r_f^2,\frac{2R_L+R_H}{3}]$ and investors roll over the debt in both asset structures. Given this, from (20) total welfare is given by

$$W(g)=\frac{R_L+R_H}{2}-\frac{8}{64}(1-\alpha)R_L-2c \tag{22}$$

for $g=\{C,U\}$ as a bank's expected probability of default at date 2 is the same in the two structures.

*Region ii.* For $\alpha_{LOW}(C)>\alpha\geq\alpha_{MID}(C)>\alpha_{LOW}(U)$, (10) is satisfied for $\rho_{12}^B(C)\in[\frac{2R_L+R_H}{3},\frac{R_L+2R_H}{3}]$ in the clustered structure and for $\rho_{12}^B(U)\in[r_f^2,\frac{2R_L+R_H}{3}]$ in the unclustered structure. Investors roll over the debt in both asset structures but the bank default probabilities now differ in the two structures. From (20) and Table 3, total welfare in the clustered structure is given by

$$W(C) = \frac{R_L + R_H}{2} - \frac{15}{64}(1-\alpha)[\frac{8}{15}R_L + \frac{3}{15}\frac{2R_L + R_H}{3}] - 2c, \tag{23}$$

and by (22) in the unclustered structure. It follows immediately that $W(U) > W(C)$.

*Regions iii1 and iii2.* For $\alpha_{MID}(C) > \alpha \geq \alpha_{LOW}(U)$, (10) cannot be satisfied for any $\rho_{12}^B(C) \leq X_i$ in the clustered structure, whereas it is still satisfied for $\rho_{12}^B(U) \in [r_f^2, \frac{2R_L+R_H}{3}]$ in the unclustered structure. Thus, the bank is liquidated and, from (21), total welfare in the clustered structure is now equal to

$$W(C) = \frac{49}{64}\left[\frac{21}{49}\frac{2R_L + R_H}{3} + \frac{21}{49}\frac{R_L + 2R_H}{3} + \frac{7}{49}R_H\right] + \frac{15}{64}r_f^2 - 2c,$$

whereas $W(U)$ is still given by (22) in the unclustered structure.

Comparing $W(C)$ and $W(U)$ gives

$$W(U) - W(C) = \frac{1}{64}[4R_H + (3+8\alpha)R_L - 15r_f^2].$$

Equating this to zero and solving for $\alpha$ as a function of $r_f^2$ gives the boundary between Regions iii1 and iii2:

$$\alpha_W = \frac{15r_f^2 - 3R_L - 4R_H}{8R_L}.$$

It can be seen that $W(U) > W(C)$ for $\alpha > \alpha_W$ and $W(U) < W(C)$ for $\alpha < \alpha_W$.

*Region iv.* For $\alpha < \alpha_{LOW}(U)$, (10) cannot be satisfied for any $\rho_{12}^B(g) \leq X_i$ so that banks are liquidated early in both structures. Total welfare is still as in (23) in the clustered structure, while, from (21), it equals

$$W(U) = \frac{39}{64}\left[\frac{13}{39}\frac{2R_L + R_H}{3} + \frac{19}{39}\frac{R_L + 2R_H}{3} + \frac{7}{39}R_H\right] + \frac{25}{64}r_f^2 - 2c$$

in the unclustered structure. The difference between the two expressions is given by

$$W(C) - W(U) = \frac{1}{32}(2R_H + 3R_L - 5r_f^2),$$

which is positive for any $r_f^2 \in [R_L, \frac{5R_L+R_H}{6}]$. $\square$

**Proof of Proposition 6**. The proposition follows immediately from the comparison of the bank expected profits $\pi_i(g)$ and $\pi_i^{LT}$ in the two structures with short-term and long-term debt. The expression for $\pi_i(g)$ is given by (11) or (12) depending on investors' rollover decisions while $\pi_i^{LT}$ is always given by (8) with $\ell = 2$. Consider the clustered structure first and the regions for investors' rollover decision in Proposition 3. In Region i, investors roll over the debt for a repayment $\rho_{12}^B(g) \in [r_f^2, \frac{2R_L+R_H}{3}]$ so that $\pi_i(C)$ is given by (11) with $q(C) = \frac{49}{64}$ and $E(X_i < \rho_{12}^B(C)|B) = \frac{8}{15}R_L$ using the conditional probabilities $\Pr(X_i = x|B)$ as in Table 2. After some simplifications, we obtain

$$\pi_i(C) - \pi_i^{LT} = r_F^2 - r_f^2$$

from which $\overline{r}_F^2(C) = r_f^2$ for all values of $r_f^2$ and $\alpha$ in Region i. In Region ii investors still roll over their debt but for $\rho_{12}^B(g) \in [\frac{2R_L+R_H}{3}, \frac{R_L+2R_H}{3}]$. The profit $\pi_i(C)$ still comes from (11) but now $E(X_i < \rho_{12}^B(C)|B) = \frac{8}{15}R_L + \frac{3}{15}\frac{2R_L+R_H}{3}$. Then,

$$\pi_i(C) - \pi_i^{LT} = r_F^2 - r_f^2 + \frac{1}{64}(1-\alpha)\frac{2R_L + R_H}{3}$$

from which $\overline{r}_F^2 = r_f^2 + \frac{1}{64}(1-\alpha)\frac{2R_L+R_H}{3}$ for all values of $r_f^2$ and $\alpha$ in Region ii. In Region iii+iv investors no longer roll over their debt. The expression for $\pi_i(C)$ is now given by (12) with $q(C) = \frac{49}{64}$ and $E(X_i > r_f^2|G) = \frac{21}{49}\frac{2R_L+R_H}{3} + \frac{21}{49}\frac{R_L+2R_H}{3} + \frac{7}{49}R_H$ using the conditional probabilities $\Pr(X_i = x|G)$ as in Table 2. After some simplifications, we obtain

$$\pi_i(C) - \pi_i^{LT} = r_F^2 - \frac{49}{64}r_f^2 + \frac{1}{64}(3R_L + 4R_H) + \frac{8}{64}\alpha R_L$$

from which $\overline{r}_F^2 = \frac{49}{64} r_f^2 + \frac{1}{64}(3R_L + 4R_H) + \frac{8}{64}\alpha R_L$ for all values of of $r_f^2$ and $\alpha$ in Regions iii+iv.

Consider now the unclustered structure. From Proposition 4 in Regions i+ii+iii investors roll over their debt and $\pi_i(U)$ is given by (11) with $q(U) = \frac{39}{64}$ and $E(X_i < \rho_{12}^B(U)|B) = \frac{8}{25}R_L$ using again the conditional probabilities $\Pr(X_i = x|B)$ as in Table 2. We then have

$$\pi_i(U) - \pi_i^{LT} = r_F^2 - r_f^2$$

from which $\overline{r}_F^2(C) = r_f^2$ for all values of $r_f^2$ and $\alpha$ in Regions i+ii+iii. In Region iv the bank is liquidated early so that $\pi_i(U)$ is given by (12) with $q(U) = \frac{39}{64}$ and $E(X_i > r_f^2|G) = \frac{13}{39}\frac{2R_L+R_H}{3} + \frac{19}{39}\frac{R_L+2R_H}{3} + \frac{7}{39}R_H$. Then,

$$\pi_i(U) - \pi_i^{LT} = r_F^2 - \frac{39}{64}r_f^2 + \frac{1}{64}(9R_L + 8R_H) + \frac{8}{64}\alpha R_L$$

from which $\overline{r}_F^2 = \frac{39}{64}r_f^2 + \frac{1}{64}(9R_L + 8R_H) + \frac{8}{64}\alpha R_L$ for all values of $r_f^2$ and $\alpha$ in Region iv. The proposition follows. $\square$

# A Derivation of the sufficiency of condition (7)

To ensure that bankruptcy only occurs when all projects in a bank's portfolio return $R_L$ for any $\ell_i = 0, .., 5$, we need to show that there exists a value of $r$ in the interval $[r_F^2, \frac{\ell_i R_L + R_H}{1+\ell_i}]$ that satisfies the investors' participation constraint (4). Substituting $\Pr(X_i < r) = (1-p)^{1+\ell_i}$ and $\Pr(X_i \geq r) = 1 - (1-p)^{1+\ell_i}$ into (4), this requires

$$(1 - (1-p)^{1+\ell_i})\frac{\ell_i R_L + R_H}{1+\ell_i} + (1-p)^{1+\ell_i}\alpha R_L \geq r_F^2 \tag{24}$$

for any $\ell_i = 0, .., 5$. When

$$\frac{\left(1-(1-p)^{1+\ell_i}\right)}{1+\ell_i}\left[R_L - \frac{(\ell_i R_L + R_H)}{1+\ell_i}\right] + (1-p)^{1+\ell_i}\log(1-p)\left[\alpha R_L - \frac{(\ell_i R_L + R_H)}{1+\ell_i}\right] \leq 0$$

then the left hand side of (24) is decreasing in $\ell_i$ for $\ell_i = 0, .., 5$, and (7) is sufficient for (24) to hold. $\square$

## B Derivation of (15) and (16)

Recall first that banks' portfolio composition in both the clustered and the unclustered structures are as given in Fig. 1. Applying the Law for the Probability of Union of Sets[7] to (14) and taking into account that bank $i$ portfolio returns $X_i = R_L$ when all three projects in its portfolio return $R_L$, we obtain

$$\begin{aligned}
1 - q(C) &= 6 \Pr\left[\bigcap_{i=1}^{3}(\theta_i = R_L)\right] - \left[6 \Pr\left(\bigcap_{i=1}^{3}(\theta_i = R_L)\right) + 9 \Pr\left(\bigcap_{i=1}^{6}(\theta_i = R_L)\right)\right] \\
&\quad + \left[2 \Pr\left(\bigcap_{i=1}^{3}(\theta_i = R_L)\right) + 18 \Pr\left(\bigcap_{i=1}^{6}(\theta_i = R_L)\right)\right] \\
&\quad - \binom{6}{4} \Pr\left(\bigcap_{i=1}^{6}(\theta_i = R_L)\right) + \binom{6}{5} \Pr\left(\bigcap_{i=1}^{6}(\theta_i = R_L)\right) - \binom{6}{6} \Pr\left(\bigcap_{i=1}^{6}(\theta_i = R_L)\right) \\
&= 2 \Pr\left[\bigcap_{i=1}^{3}(\theta_i = R_L)\right] - \Pr\left[\bigcap_{i=1}^{6}(\theta_i = R_L)\right]. \qquad (25)
\end{aligned}$$

in the clustered structure, and

$$\begin{aligned}
1 - q(U) &= 6 \Pr\left[\bigcap_{i=1}^{3}(\theta_i = R_L)\right] - \left[6 \Pr\left(\bigcap_{i=1}^{4}(\theta_i = R_L)\right) + 6 \Pr\left(\bigcap_{i=1}^{5}(\theta_i = R_L)\right)\right. \\
&\quad \left. +3 \Pr\left(\bigcap_{i=1}^{6}(\theta_i = R_L)\right)\right] + \left[6 \Pr\left(\bigcap_{i=1}^{5}(\theta_i = R_L)\right) + 14 \Pr\left(\bigcap_{i=1}^{6}(\theta_i = R_L)\right)\right] \\
&\quad - \binom{6}{4} \Pr\left(\bigcap_{i=1}^{6}(\theta_i = R_L)\right) + \binom{6}{5} \Pr\left(\bigcap_{i=1}^{6}(\theta_i = R_L)\right) - \binom{6}{6} \Pr\left(\bigcap_{i=1}^{6}(\theta_i = R_L)\right) \\
&= 6 \Pr\left[\bigcap_{i=1}^{3}(\theta_i = R_L)\right] - 6 \Pr\left[\bigcap_{i=1}^{4}(\theta_i = R_L)\right] + \Pr\left[\bigcap_{i=1}^{6}(\theta_i = R_L)\right] \qquad (26)
\end{aligned}$$

[7] This states that $\Pr(\cup_{i=1}^{m} A_i) = \Sigma_i \Pr(A_i) - \Sigma_{ij} \Pr(A_i \cap A_j) + \Sigma_{ijk} \Pr(A_i \cap A_j \cap A_k) - ... + (-1)^m \Pr(\cap_i A_i)$, for any set of events $A_i$ (see http://mathworld.wolfram.com/Probability.html).

in the unclustered structure. It remains to show that

$$\Pr\left[\bigcap_{i=1}^{n}(\theta_i = R_L)\right] = \sum_{m=n}^{6} \frac{\binom{6-n}{6-m}}{2^6}. \tag{27}$$

for any $n \in \{3, 4, 6\}$ where $m \leq 6$ is the number of projects returning $R_L$ in the combination $mR_L, (6-m)R_H$. To see this, we first make use of the Law of Total Probabilities[8] and obtain

$$\begin{aligned} \Pr\left[\bigcap_{i=1}^{n}(\theta_i = R_L)\right] &= \sum_{m=0}^{6} \Pr\left[\bigcap_{i=1}^{n}(\theta_i = R_L)\middle|(mR_L,(6-m)R_H)\right]\Pr(mR_L,(6-m)R_H) \\ &= \sum_{m=n}^{6} \Pr\left[\bigcap_{i=1}^{n}(\theta_i = R_L)\middle|(mR_L,(6-m)R_H)\right]\Pr(mR_L,(6-m)R_H) \end{aligned} \tag{28}$$

once we take into account $\Pr\left[\bigcap_{i=1}^{n}(\theta_i = R_L)\middle|(mR_L,(6-m)R_H)\right] = 0$ for any $m < n$. Then, for each combination of projects $(mR_L, (6-m)R_H)$, there are $\binom{6-n}{6-m}$ ways of selecting $n \leq m$ projects that return $R_L$. This implies that

$$\Pr\left[\bigcap_{i=1}^{n}(\theta_i = R_L)\middle|(mR_L,(6-m)R_H)\right] = \frac{\binom{6-n}{6-m}}{\binom{6}{m}}. \tag{29}$$

Since $\Pr(mR_L, (6-m)R_H) = \frac{\binom{6}{m}}{2^6}$ , (27) follows immediately. Using (27) in (25) and (26) gives (15) and (16).

## C Derivation of Tables 2 and 3

Consider Table 2 for the clustered structure first. Clearly there are no default states when $m \leq 2$ as in states 1, 2 and 3. From (15) it follows that for any $m = \{3, 4, 5\}$ the number of default states is $2\binom{6-3}{6-m}$ out of the $\binom{6}{m}$ states where the combination $(mR_L, (6-m)R_H)$

[8] This states that given $n$ mutually exclusive events $A_1$, $A_2$, ..., $A_n$ with probabilities summing to 1, then $\Pr(B) = \Sigma_i \Pr(B/A_i)\Pr(A_i)$ where $B$ is an arbitrary event (see http://mathworld.wolfram.com/TotalProbabilityTheorem.html).

is realized. For example, given $(3R_L, 3R_H)$ (that is, $m = 3$) the number of default states equals $2\binom{6-3}{6-3} = 2$. In each of the 2 default states, 3 banks in one cluster will default with a portfolio return of $R_L$ while the 3 other banks will remain solvent with a portfolio returning $R_H$. Thus, bank $i$'s portfolio returns $X_i = R_L$ in 1 state and $X_i = R_H$ in the other state out of the 2 default states. Similar considerations hold for the other entries. With $m = 6$ there is clearly only one default state where all banks have $X_i = R_L$, which can be also derived from $2\binom{6-3}{6-6} - 1$ in (15). The number of no default states when $S = G$ is simply the difference between the $\binom{6}{m}$ states where the combination $(mR_L, (6-m)R_H)$ is realized and the number of default states. The distribution for $X_i$ conditional on $S = G$ can be found similarly to before. For example, given $(3R_L, 3R_H)$, there are $\binom{6}{3} - 2\binom{6-3}{6-m} = 18$ no default states where the good signal arrives. In such states, 3 banks in one cluster will have a portfolio returning $\frac{2R_L+R_H}{3}$ while the other 3 banks will have $X_i = \frac{R_L+2R_H}{3}$. Thus, bank $i$'s portfolio returns $X_i = \frac{2R_L+R_H}{3}$ in 9 states and $X_i = \frac{R_L+2R_H}{3}$ in the other 9 states out of the 18 no default states. Similar considerations hold for the other entries. The last row of Table 2 indicates that, for example, out of the 15 total default states, bank $i$ has portfolio return $X_i = R_L$ in 8 states; and out of the 49 states where no defaults occur its portfolio returns $X_i = \frac{2R_L+R_H}{3}$ in 21 states. Similarly for the other returns conditional on $S = B, G$.

The conditional distribution in the unclustered structure as described in Table 3 is derived similarly. The number of default states given the combination $(mR_L, (6-m)R_H)$ follows from (16). The bad signal arrives in $6\binom{6-3}{6-m}$ states for $m = 3$; in $6\binom{6-3}{6-m} - 6\binom{6-4}{6-m}$ states for $m \in \{4, 5\}$ and in $6\binom{6-3}{6-m} - 6\binom{6-4}{6-m} + 1$ for $m = 6$ out of the $\binom{6}{m}$ states where the combination $(mR_L, (6-m)R_H)$ is realized. As before, for given default states the conditional distribution of $X_i$ can easily be derived. For example, given $(3R_L, 3R_H)$ (that is, $m = 3$) there are $6\binom{6-3}{6-3} = 6$ default states where bank $i$'s portfolio return is $X_i = R_L$ or $X_i = R_H$ in 1 state each, and $X_i = \frac{2R_L+R_H}{3}$ or $X_i = \frac{R_L+2R_H}{3}$ in 2 states each. Similarly for the other entries conditional on the bad signal. The number of no default states is again derived as the difference between the $\binom{6}{m}$ states where the combination $(mR_L, (6-m)R_H)$

is realized and the number of default states. For example, given $(3R_L, 3R_H)$, there are $\binom{6}{3} - 6\binom{6-3}{6-3} = 14$ no default states where the good signal arrives. In such states, bank $i$'s portfolio returns $X_i = \frac{2R_L+R_H}{3}$ in 7 states and $X_i = \frac{R_L+2R_H}{3}$ in the other 7 states out of the 14 no default states. Similar considerations hold for the other entries. The last row in Table 3 indicates that out of the 25 total default states, bank $i$ has portfolio return $X_i = R_L$ in 8 states; and out of the 39 states where no defaults occur its portfolio returns $X_i = \frac{2R_L+R_H}{3}$ in 13 states. Similarly for the other returns conditional on $S = B, G$.

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Table 1: Project realizations and distribution of bank $i$'s portfolio return $X_i$

| | Project realizations | Number of states | Bank $i$'s return $X_i$ | | | |
|---|---|---|---|---|---|---|
| | | | $R_L$ | $\frac{2R_L+R_H}{3}$ | $\frac{R_L+2R_H}{3}$ | $R_H$ |
| 1 | $6R_H$ | $\binom{6}{0}=1$ | 0 | 0 | 0 | 1 |
| 2 | $R_L, 5R_H$ | $\binom{6}{1}=6$ | 0 | 0 | 3 | 3 |
| 3 | $2R_L, 4R_H$ | $\binom{6}{2}=15$ | 0 | 3 | 9 | 3 |
| 4 | $3R_L, 3R_H$ | $\binom{6}{3}=20$ | 1 | 9 | 9 | 1 |
| 5 | $4R_L, 2R_H$ | $\binom{6}{4}=15$ | 3 | 9 | 3 | 0 |
| 6 | $5R_L, R_H$ | $\binom{6}{5}=6$ | 3 | 3 | 0 | 0 |
| 7 | $6R_L$ | $\binom{6}{6}=1$ | 1 | 0 | 0 | 0 |
| | Total | 64 | 8 | 24 | 24 | 8 |

Table 2: Distribution of bank $i$'s portfolio return $X_i$ in the *clustered* structure

with short-term finance conditional on the signal

| | Project | Default states | Bank $i$'s return $X_i$ given $s = B$ | | | | No default states | Bank $i$'s return $X_i$ given $s = G$ | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | realizations | given $s = B$ | $R_L$ | $\frac{2R_L+R_H}{3}$ | $\frac{R_L+2R_H}{3}$ | $R_H$ | given $s = G$ | $R_L$ | $\frac{2R_L+R_H}{3}$ | $\frac{R_L+2R_H}{3}$ | $R_H$ |
| 1 | $6R_H$ | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 |
| 2 | $R_L, 5R_H$ | 0 | 0 | 0 | 0 | 0 | 6 | 0 | 0 | 3 | 3 |
| 3 | $2R_L, 4R_H$ | 0 | 0 | 0 | 0 | 0 | 15 | 0 | 3 | 9 | 3 |
| 4 | $3R_L, 3R_H$ | 2 | 1 | 0 | 0 | 1 | 18 | 0 | 9 | 9 | 0 |
| 5 | $4R_L, 2R_H$ | 6 | 3 | 0 | 3 | 0 | 9 | 0 | 9 | 0 | 0 |
| 6 | $5R_L, R_H$ | 6 | 3 | 3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 7 | $6R_L$ | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Total | | 15 | 8 | 3 | 3 | 1 | 49 | 0 | 21 | 21 | 7 |

Table 3: Distribution of bank $i$'s portfolio return $X_i$ in the *unclustered* structure with short-term finance conditional on the signal

| | Project realizations | Default states given $s = B$ | Bank $i$'s return $X_i$ given $s = B$ | | | | No default states given $s = G$ | Bank $i$'s return $X_i$ given $s = G$ | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | $R_L$ | $\frac{2R_L+R_H}{3}$ | $\frac{R_L+2R_H}{3}$ | $R_H$ | | $R_L$ | $\frac{2R_L+R_H}{3}$ | $\frac{R_L+2R_H}{3}$ | $R_H$ |
| 1 | $6R_H$ | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 |
| 2 | $R_L, 5R_H$ | 0 | 0 | 0 | 0 | 0 | 6 | 0 | 0 | 3 | 3 |
| 3 | $2R_L, 4R_H$ | 0 | 0 | 0 | 0 | 0 | 15 | 0 | 3 | 9 | 3 |
| 4 | $3R_L, 3R_H$ | 6 | 1 | 2 | 2 | 1 | 14 | 0 | 7 | 7 | 0 |
| 5 | $4R_L, 2R_H$ | 12 | 3 | 6 | 3 | 0 | 3 | 0 | 3 | 0 | 0 |
| 6 | $5R_L, R_H$ | 6 | 3 | 3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 7 | $6R_L$ | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Total | | 25 | 8 | 11 | 5 | 1 | 39 | 0 | 13 | 19 | 7 |

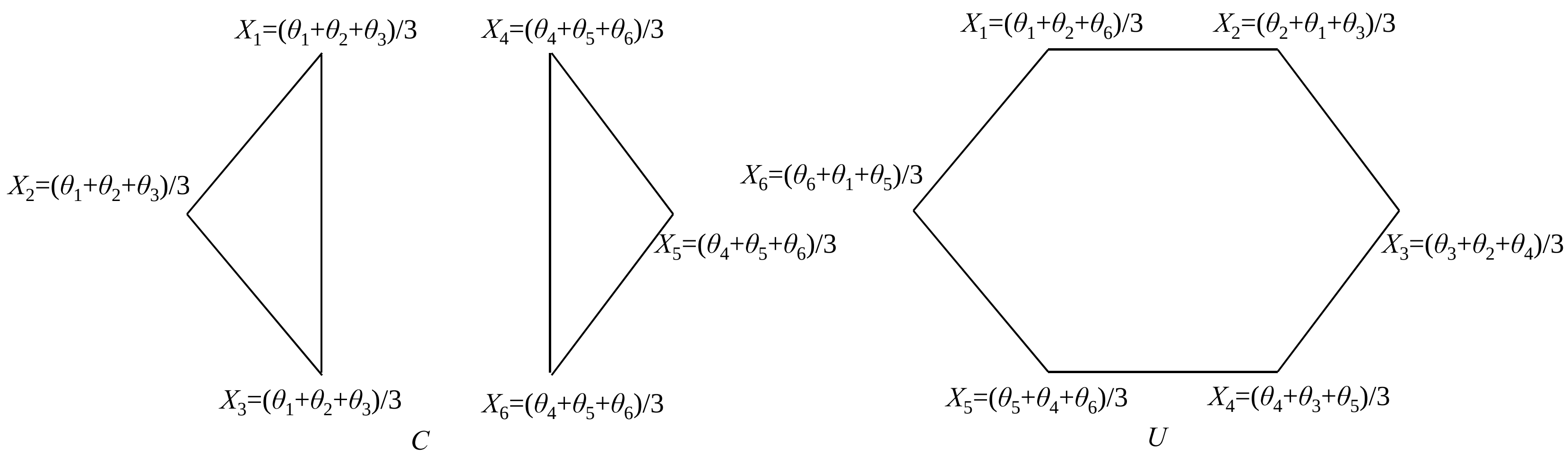


**Fig. 1: Clustered (C) and unclustered (U) asset structures**

The figure depicts the clustered (C) and the unclustered (U) asset structures. In the former, banks are connected in two clusters of three banks each. Within each cluster, banks hold identical portfolios with return $X_i$ but the two clusters are independent of each other. In the latter, banks are all connected in a circle. Each of them exchanges projects only with the two neighboring banks and none of the banks holds identical portfolios.

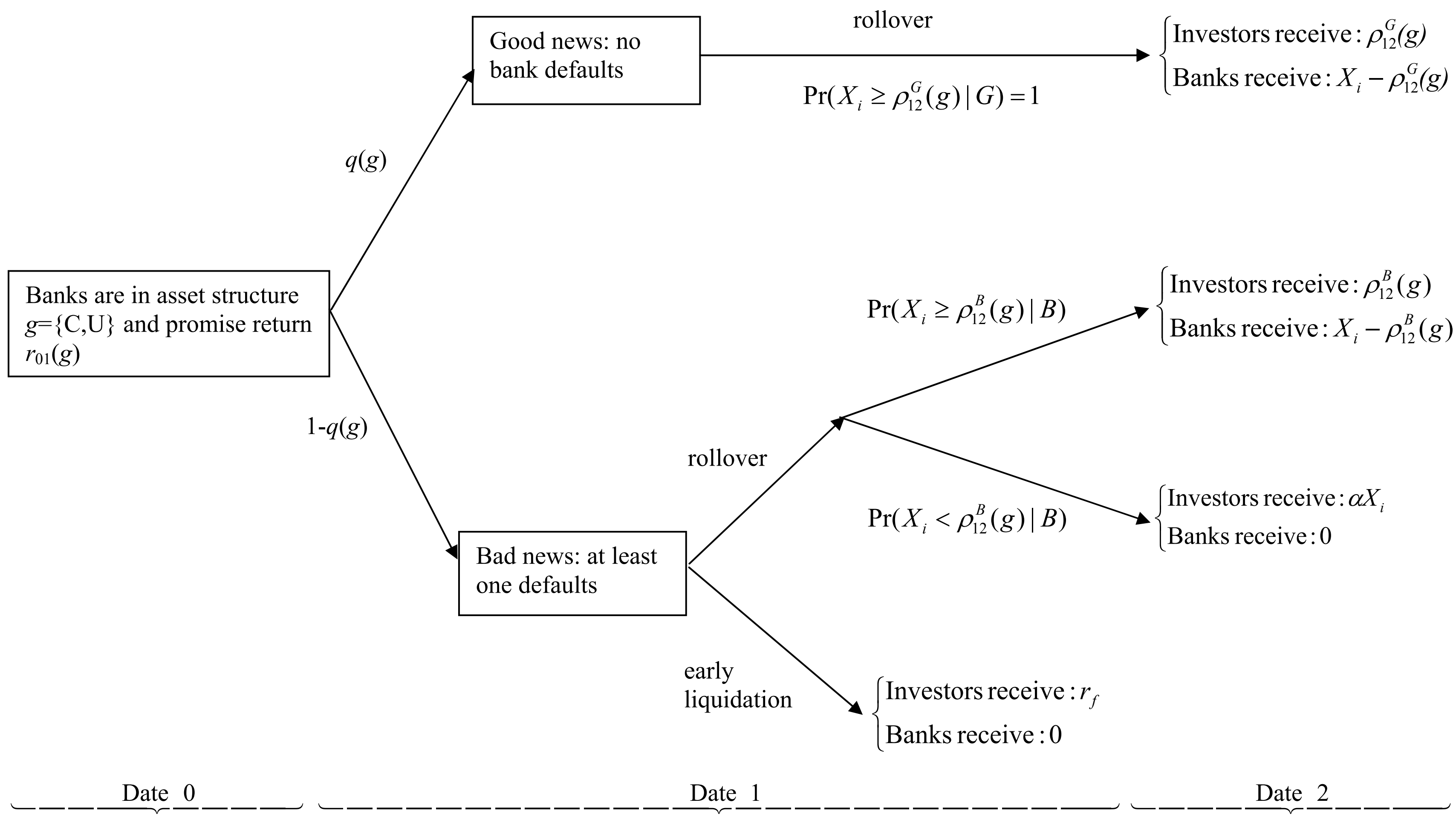


**Fig. 2: Sequence of events with short-term finance**

The figure shows the timing of the model with short-term finance. At date 0 each bank in the asset structure g ={ C,U} raises one unit of funds in exchange for a promised return $r_{01}(g)$ at date 1. At the beginning of date 1, before investors are repaid, a signal $S = \{G, B\}$ is realized. With probability $q(g)$, it brings the good news that all banks will be solvent at date 2. With probability 1-$q(g)$, it brings the bad news that at least one bank will default at date 2. Investors decide whether to retain $r_{01}(g)$ or roll it over for a total promised repayment of $\rho_{12}^S(g)$ at date 2. When the debt is rolled over, the bank continues till date 2. If it remains solvent, which occurs with probability $\Pr(X_i \geq \rho_{12}^S(g)|\mathrm{B})$, investors receive $\rho_{12}^S(g)$ and the bank $X_i$ - $\rho_{12}^S(g)$. If the bank defaults at date 2, which occurs with probability $\Pr(X_i < \rho_{12}^S(g)|\mathrm{B})$, investors receive $\alpha X_i$ and the bank zero. When the debt is not rolled over, the bank is forced into early liquidation at date 1. Investors obtain $r_f$ and the bank zero.

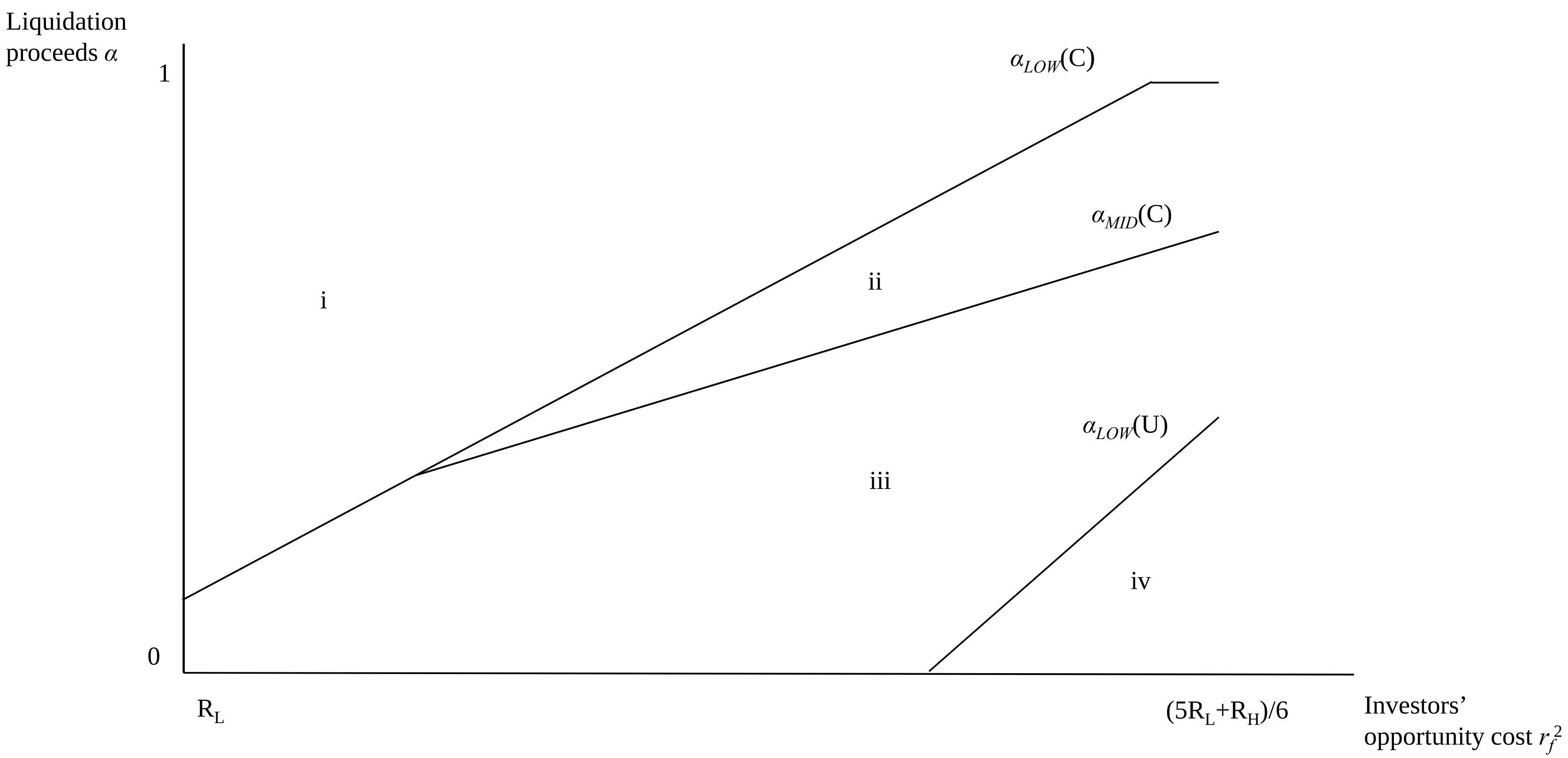


**Fig. 3: Investors' rollover decision in the clustered and unclustered asset structures with short-term finance when the bad signal arrives**

The figure depicts investors' rollover decision with short-term finance in both structures when the bad signal arrives as a function of the investors' opportunity cost $r_f^2$ and of the fraction $\alpha$ of the bank's portfolio return that investors receive in case of default. In Region i debt is rolled over for a repayment $\rho_{12}^B(g) \in (r_f^2, (2R_L + R_H)/3]$ in both structures. In Region ii rollover occurs still in both structures but in the clustered structure the repayment is now $\rho_{12}^B(g) \in ((2R_L + R_H)/3, (R_L + 2R_H)/3]$. In Region iii debt is rolled over in the unclustered structure but not in the clustered one. In Region iv rollover does not occur in either asset structure. The expressions for the boundaries $\alpha_{LOW}$(C), $\alpha_{MID}$(C), and $\alpha_{LOW}$(U) are provided in the Appendix in the proof of Propositions 3 and 4.

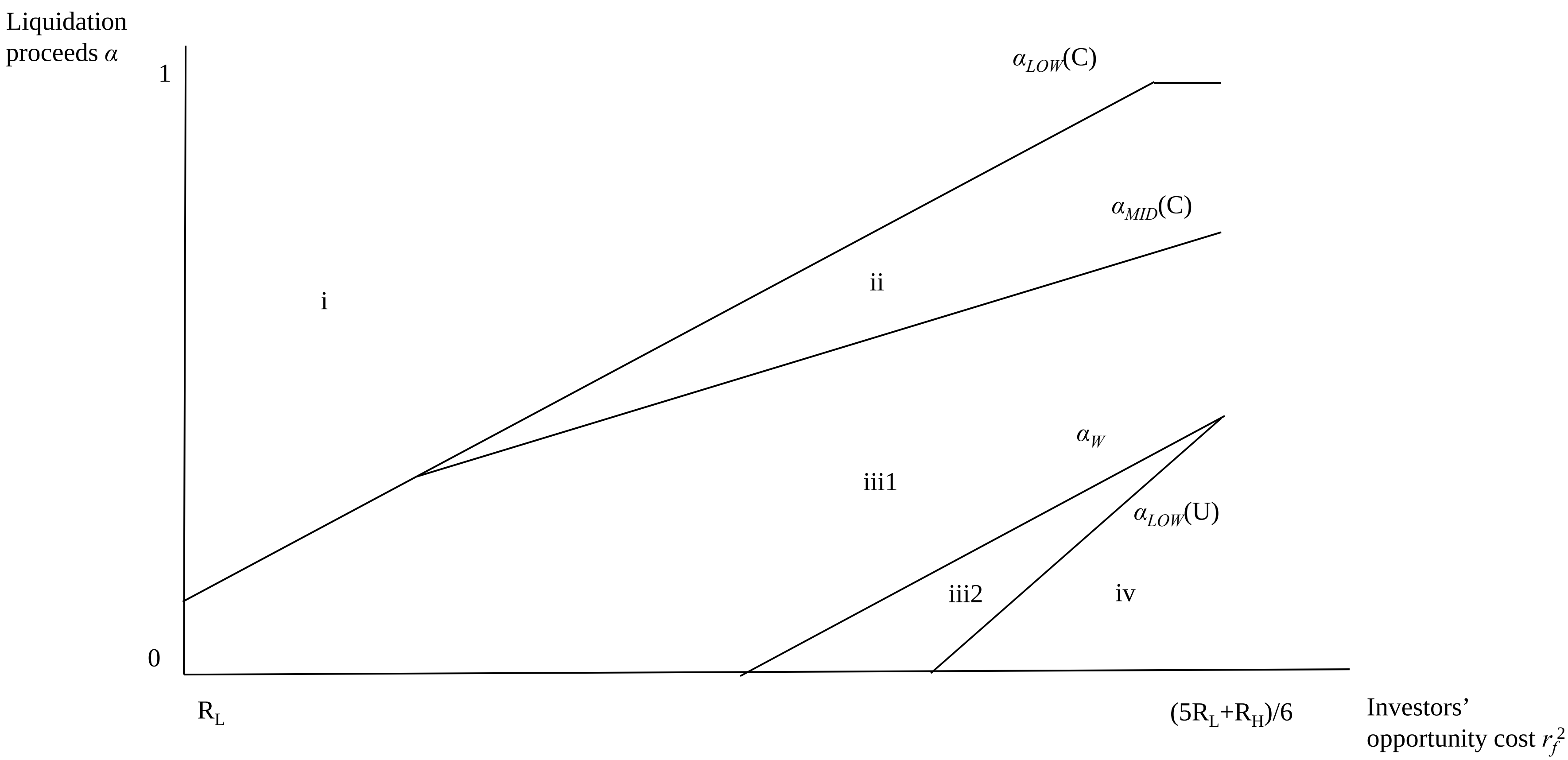


**Fig. 4: Total welfare in the clustered and unclustered asset structures with short-term finance**

The figure depicts total welfare in the clustered and unclustered structures as a function of the investors' opportunity cost $r_f^2$ and of the fraction $\alpha$ of the bank's portfolio return that investors receive in case of default. In Region i, total welfare is the same in both structures. In Region ii+iii1, total welfare is higher in the unclustered structure. In Region iii2+iv, total welfare is higher in the clustered structure. The expressions for $\alpha_{LOW}$(C), $\alpha_{MID}$(C), $\alpha_{LOW}$(U), and $\alpha_W$ are provided in the Appendix in the proof of Propositions 3, 4 and 5.

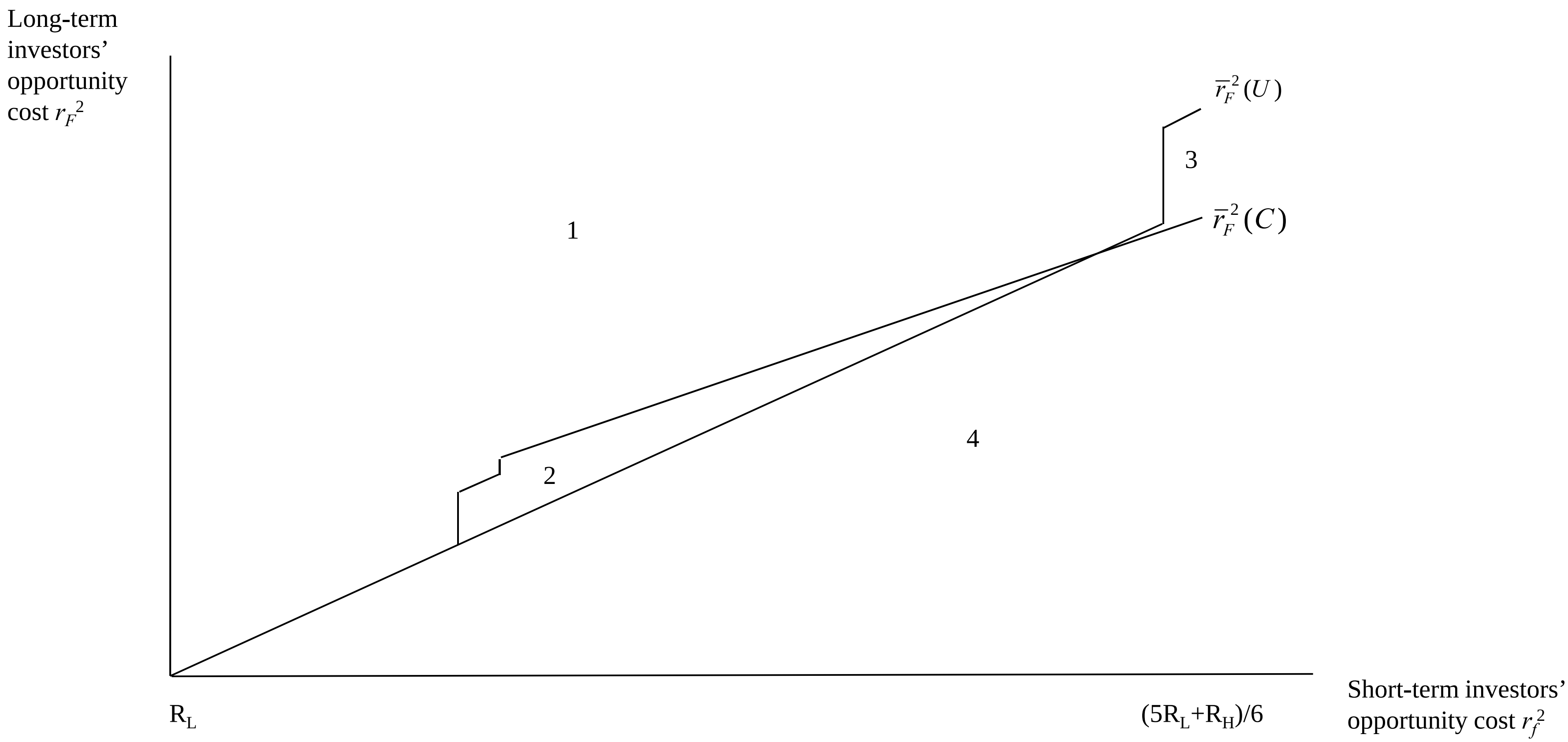


**Fig. 5: Bank's choice of debt maturity structure**

The figure plots the bank's choice of debt maturity structure as a function of the long-term investors' opportunity cost $r_F^2$ and the short-term investors' opportunity cost $r_f^2$ for a given value of the fraction $\alpha$ of the bank's portfolio return that investors receive in case of default. In Region 1 the bank finds it optimal to raise short-term debt in both the clustered and the unclustered structures. In Region 2, short-term debt is optimal in the unclustered structure while long-term debt is optimal in the clustered structure. In Region 3 the opposite happens, and short-term debt is optimal in the clustered structure only. In Region 4 long-term debt is optimal in both structures.