Trading and Information Diffusion in Over-the-Counter Markets*

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Abstract

We propose a model of trade in OTC markets in which each dealer with private information can engage in bilateral transactions with other dealers, determined by her links in a network. Each dealer's strategy is represented as a quantity-price schedule. We analyze the effect of trade decentralization and adverse selection on information diffusion, expected profits, trading costs and welfare. Information diffusion through prices is not affected by dealers' strategic trading motives, and there is an informational externality constraining the informativeness of prices. Trade decentralization can both increase or decrease welfare. The main determinant of a dealer's trading cost is the centrality of her counterparties. Central dealers tend to learn more, trade more at lower costs and earn higher expected profit.

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1 Introduction

A vast proportion of financial assets is traded in over-the-counter (OTC) markets. In these markets, transactions are bilateral, prices are dispersed, trading relationships are persistent, and, typically, a small number of large dealers intermediate a large share of the trading volume. In this paper, we explore a novel approach to modeling OTC markets that reflects these features.

In our model, each dealer with private information can engage in several bilateral transactions with her potential trading partners, determined by her links in a network. Each dealer's strategy is represented as a quantity-price schedule. Our focus is on how decentralization (characterized by the structure of the dealer network) and adverse selection jointly influence information diffusion, expected profits, trading costs, and welfare. We prove that information diffusion through prices is not affected by strategic considerations in a well-defined sense. We show that each equilibrium price depends on all the information available in the economy, incorporating even the signals of dealers located far from a given transaction. We identify an informational externality constraining the informativeness of prices. We highlight that decentralization per se can both increase or decrease welfare and that the main determinant of a dealer's trading cost is not her centrality but the centrality of her counterparties. By extensive simulation we show that more central dealers tend to learn more, trade more at lower costs, and earn higher expected profit. However, we also explain why in some special cases, more connected dealers might earn a lower expected profit.

In our main specification, there are n risk-neutral dealers organized in a dealer network. Intuitively, a link between i and j represents that they are potential counterparties in a trade. There is a single risky asset in zero net supply. The final value of the asset is uncertain and interdependent across dealers with an arbitrary correlation coefficient controlling the relative importance of the common and private components. Each dealer observes a private signal about her value, and all dealers have the same quality of information. Since values are interdependent, it is valuable to infer each other's signals. Values and signals are drawn from a known multivariate normal distribution. Each dealer simultaneously chooses her trading strategy, understanding her price effect given other dealers' strategies. For any private signal, each dealer's trading strategy is a generalized demand function that specifies the quantity of the asset she is willing to trade with each of her counterparties, depending on the vector of prices in the transactions she engages in. Each dealer, in addition to trading with other dealers, also trades with price-sensitive customers. In equilibrium, prices and quantities have to be consistent with the set of generalized demand functions and the market clearing conditions for each link. We refer to this structure as the OTC game. The OTC game is, essentially, a generalization of the Vives (2011) variant of Kyle (1989) to networks. We consider general, connected networks.

We show that equilibrium beliefs in the OTC game are independent of dealers' strategic considerations. In fact, we construct a separate game, in which dealers do not trade, that generates the same posterior beliefs. In this simpler, auxiliary game, dealers are connected in the same network and acting in the same informational environment as in the OTC game. However, dealers' aim is to make a best guess of their own value conditional on their signals and the guesses of the other dealers they are connected to. We label this structure the conditional guessing game. As each dealer's equilibrium guess depends on her neighbors' guesses, and through those, on her neighbors' neighbors' guesses etc., each equilibrium guess will partially incorporate the private information of all the dealers in a connected network. However, dealers do not internalize how the informativeness of their guess affects others' decisions, and the equilibrium is typically not be informationally efficient. That is, dealers tend to put too much weight on their own signal, making their guess inefficiently informative about the common component.

In the OTC game, we show that each equilibrium price is a weighted sum of the posterior beliefs of the counterparties that participate in the transaction, and, hence, it inherits the main properties of beliefs. In addition, each dealer's equilibrium position is proportional to the difference between her expectation and the price. Therefore, a dealer tends to sell at a price higher than her belief to relatively optimistic counterparties and buys at a price lower than her belief from pessimists. This gives rise to dispersed prices and profitable intermediation for dealers with many counterparties, as it is characteristic of real-world OTC markets. The proportionality coefficient of a dealer's position is the inverse of her price impact in that transaction. In turn, the dealer's price impact is smaller if her counterparty is less concerned about adverse selection, either because the common value component is less important, or because she is more central and learns from several other prices.

To gain further insights on our main topics, we proceed in two distinct ways. First, we use the network formation model introduced by Jackson and Rogers (2007) to generate random networks that are calibrated to match the European CDS market, as described by Brunnermeier, Clerc and Scheicher (2013). We illustrate in a wide range of realistic networks that have a core-periphery structure that we should expect a more connected dealer to learn more, intermediate more, trade larger gross volume at lower price impact, and make more profit. We contrast these predictions with findings in the empirical literature.

Second, we gain further insights on welfare, expected profits, and illiquidity by analyzing trade in the most common OTC network structures. In particular, we isolate the effect of decentralization by comparing the complete OTC network with centralized markets; we illustrate the role of link density by comparing circulant OTC networks in which we successively increase the number of links each dealer has; and we analyze the effect of asymmetric number of links in the star OTC network. We show that centralized trading might not improve welfare and, explain that, for certain parameters, more links imply more profits only when the network exhibits assortativity.

Finally, we generalize our model to study market segmentation by allowing multiple dealers at each node of the dealer network. For example, by considering a star network with n nodes, we can model an economy with $(n-1)$ trading venues where only one central group trades in all venues, while each of the other $(n-1)$ periphery groups trade in only one trading venue. Numerically, we show that dealers in periphery groups might face less illiquidity, as captured by a lower price impact, and might learn more in a more segmented market

Related literature

Most models of OTC markets are based on search and bargaining (e.g., Duffie, Garleanu and Pedersen (2005); Duffie, Gârleanu and Pedersen (2007); Lagos, Rocheteau and Weill (2008); Vayanos and Weill (2008); Lagos and Rocheteau (2009); Afonso and Lagos (2012); and Atkeson, Eisfeldt and Weill (2012)). By construction, in search models transactions are between atomistic dealers through non-persistent links. Therefore, our approach is more suitable to capture effects of high market concentration implied by the presence of few large dealers intermediating the vast proportion of volume. At the same time, we collapse trade to a single period missing implications on the dynamic dimension. In this sense, we view these approaches to be complementary. However, models of learning through trade based on search require non-standard structures and are hard to compare to existing results on centralized markets (e.g., Duffie, Malamud and Manso (2009); Golosov, Lorenzoni and Tsyvinski (2009)).[1] Our approach is compatible with the standard, jointly normal framework of asymmetric information and learning.

There is a growing literature studying trading in a network (e.g., Kranton and Minehart (2001); Rahi and Zigrand (2006); Gale and Kariv (2007); Gofman (2011); Condorelli and Galeotti (2012); Choi, Galeotti and Goyal (2013); Malamud and Rostek (2013); Manea (2013); Nava (2013)). These papers typically consider either the sequential trade of a single unit of the asset or a Cournot-type quantity competition.[2] In contrast, we allow agents to form (generalized) demand schedules conditioning the quantities for each of their transactions on the vector equilibrium prices in these transactions. This emphasizes that the terms of the various transactions of a dealer are interconnected in an OTC market. Also, to our knowledge, none of the papers within this class addresses the issue of information aggregation which is the focus of our analysis.[3]

A separate literature studies Bayesian (Acemoglu et al. (2011)) and non-Bayesian (Bala and Goyal (1998); DeMarzo, Vayanos and Zwiebel (2003); Golub and Jackson (2010)) learning in the context of arbitrary connected social networks. In these papers, agents update their beliefs about a payoff-relevant state after observing the actions of their neighbors in the network. Our model complements these works by considering that (Bayesian) learning takes place through trading.

The paper is organized as follows. The following section introduces the model set-up and the equilibrium concept. In Section 3, we derive the equilibrium, and give sufficient conditions for existence. We characterize the informational content of prices and characteristics of information diffusion in Section 4. In Section 5, we study expected profit, welfare, and illiquidity based on some of the most common examples. We extend our analysis to simulated random networks in Section 6. In Section 7, we show how our framework can be extended to study market

[1] The main focus of these models is the time-dimension of information diffusion across agents. In these models, incentives to share information and to learn are driven by the fact that two agents meet repeatedly or any agent meet with counterparties of their counterparties with zero probability. This is in contrast with our approach where delears understand that the network structure may lead to overlapping information among their counterparties.

[2] As an exception, Malamud and Rostek (2013) also use a multi-unit double-auction set-up to model a decentralized market. However, they do not consider the problem of learning through trade.

[3] While there is another stream of papers (e.g., Ozsoylev and Walden (2011); Colla and Mele (2010); Walden (2013)) that consider that traders have access to the information of their neighbors in a network, in these models trade takes place in a centralized market.

segmentation. In Section 8, we conclude.

2 A General Model of Trading in OTC Markets

2.1 The model set-up

We consider an economy with n risk-neutral dealers that trade bilaterally a divisible risky asset. All trades take place at the same time. Dealers, apart from trading with each other, also serve a price sensitive customer-base. Each dealer is uncertain about the value of the asset. This uncertainty is captured by θ^i, referred to as dealer i's value. We consider that values are interdependent across dealers. In particular, the value of the asset for dealer i can be explained by a component, $\hat{\theta}$, that is common to all dealers, and a component, η^i, that is specific to dealer i, such that

$$\theta^i = \hat{\theta} + \eta^i,$$

with $\hat{\theta} \sim N(0, \sigma^2_{\hat{\theta}})$, $\eta^i \sim IID\, N(0, \sigma^2_\eta)$, and $\mathcal{V}(\hat{\theta}, \eta^i) = 0$, where $\mathcal{V}(\cdot, \cdot)$ represents the variance-covariance operator. This implies that θ^i is normally distributed with mean 0 and variance $\sigma^2_\theta = \sigma^2_{\hat{\theta}} + \sigma^2_\eta$. Differences in dealers' values reflect, for instance, differences in usage of the asset as collateral, in technologies to repackage and resell cashflows, in risk-management constraints. The degree of the interdependence between dealers' values is captured by the correlation coefficient

$$\rho = \frac{\sigma^2_{\hat{\theta}}}{\sigma^2_\theta},$$

where $\rho \in [0, 1]$. This representation is useful as we can vary the degree of interdependence, ρ, while keeping the variance σ^2_θ constant.

The asset is in zero net-supply. This is without loss of generality, provided supply is constant. We do not assume any constraints on the size or sign of dealers' positions.

We assume that each dealer receives a private signal, s^i, such that

$$s^i = \theta^i + \varepsilon^i,$$

where $\varepsilon^i \sim IID\, N(0, \sigma^2_\varepsilon)$ and $\mathcal{V}(\theta^j, \varepsilon^i) = 0$, for all i and j.

Dealers are organized into a trading network, g. A link $ij \in g$ implies that i and j are potential trading partners, or neighbors in the network g. Intuitively, agent i and j know and sufficiently trust each other to trade if they find mutually agreeable terms. Let g^i denote the set of i's neighbors and $m^i \equiv |g^i|$ the number of i's neighbors. If two dealers have a link, let q^i_{ij} denote the quantity that dealer i trades over link ij. The price at which trade takes place is denoted by p_{ij}. Links in the network are undirected, such that if $ij \in g$, then $ji \in g$ as well. The notation reflects this property as $p_{ij} = p_{ji}$ and $q^i_{ij} = q^i_{ji}$, for instance.

While our main results hold for any network, throughout the paper, we illustrate the results using two main types of networks as examples.

Example 1 *In an (n, m) **circulant network,** with n odd and $m < n$ even, if dealers are arranged in a ring then each dealer is connected with $m/2$ other dealers on her left and $m/2$ on her right. The $(n, 2)$ circulant network is the circle, while the $(n, n-1)$ circulant network is the complete network.*

Example 2 *In an star **network**, one dealer is connected with $n-1$ other dealers, and no other links exist.*

We define a one-shot game where each dealer chooses an optimal trading strategy, provided she takes as given others' strategies but she understands that her trade has a price effect. In particular, the strategy of a dealer i is a map from the signal space to the space of *generalized demand functions*. For each dealer i with signal s^i, a generalized demand function is a continuous function $\mathbf{Q}^i : R^{m^i} \to R^{m^i}$ which maps the vector of prices[4], $\mathbf{p}_{g^i} = (p_{ij})_{j \in g^i}$, that prevail in the transactions that dealer i participates in network g into a vector of quantities she wishes to trade with each of her counterparties. The j-th element of this correspondence, $Q^i_{ij}(s^i; \mathbf{p}_{g^i})$, represents her demand function when her counterparty is dealer j, such that

$$\mathbf{Q}^i(s^i; \mathbf{p}_{g^i}) = \left(Q^i_{ij}(s^i; \mathbf{p}_{g^i})\right)_{j \in g^i}.$$

Note that a dealer can buy a given quantity at a given price from one counterparty and sell a different quantity at a different price to another at the same time. Also, the quantity that dealer i trades with dealer j, $q^i_{ij} = Q^i_{ij}(s^i; \mathbf{p}_{g^i})$, depends on all the prices $\mathbf{p}_{g^i}$. For example, if k is linked to i who is linked to j, a high demand from dealer k might raise the bilateral price

[4]A vector is always considered to be a column vector, unless explicitly stated otherwise.

p_{ki}. This might make dealer i to revise her estimation of her value upwards and adjust her supplied quantity both to k and to j accordingly. However, $Q^i_{ij}(s^i; \mathbf{p}_{g^i})$ depends only on $\mathbf{p}_{g^i}$ but not on the full price vector. This emphasizes a critical feature of OTC markets, namely that the price and the quantity traded in a bilateral transaction are known only by the two counterparties involved in the trade and not immediately revealed to all market participants. While OTC trading protocols do not typically involve the submission of full demand schedules, we think of generalized demand functions as a reduced-form price determination mechanism that captures the repeated exchange of limit and market orders (i.e., the offer and acceptance of quotes) across fixed counterparties that have persistent links, within a short time-interval. To illustrate this mapping, we explicitly model the price-discovery process in Appendix D. This also shows why our specification need not rely on the implicit assumption of a Walrasian auctioneer.

Apart from trading with each other, each dealer also serves a price-sensitive customer base. Customers have quadratic preferences for holding a quantity q of the asset. We assume that a dealer i uses each link ij to satisfy an exogenously given fraction of her customer base. In particular, we consider that dealer i trades with the customers she associates to the link ij at the same price she trades with dealer j, p_{ij}, plus a fixed, exogenous, markup. This implies that for each transaction between i and j, the customer base generates a downward-sloping demand

$$D_{ij}(p_{ij}) = \beta_{ij} p_{ij}, \tag{1}$$

where the constant $\beta_{ij} < 0$ is a summary statistics for dealer i and j's customers' preferences, as well as the markup the dealers charge. This specification captures in reduced form the relationship between customers and dealers in OTC markets.

The expected payoff for dealer i corresponding to the strategy profile $\left\{\mathbf{Q}^i\left(s^i; \mathbf{p}_{g^i}\right)\right\}_{i\in\{1,\ldots,n\}}$ is

$$E\left[\sum_{j\in g^i} Q^i_{ij}(s^i; \mathbf{p}_{g^i})\left(\theta^i - p_{ij}\right)|s^i, \mathbf{p}_{g^i}\right], \tag{2}$$

where p_{ij} are the elements of the bilateral clearing price vector $\mathbf{p}$ defined by the smallest element of the set

$$\widetilde{\mathbf{P}}\left(\left\{\mathbf{Q}^i\left(s^i;\mathbf{p}_{g^i}\right)\right\}_i,\mathbf{s}\right)\equiv\left\{\mathbf{p}\;\middle|\;Q^i_{ij}\left(s^i;\mathbf{p}_{g^i}\right)+Q^j_{ij}\left(s^j;\mathbf{p}_{g^j}\right)+\beta_{ij}p_{ij}=0,\,\forall\,ij\in g\right\} \tag{3}$$

by lexicographical ordering[5], if $\widetilde{\mathbf{P}}$ is non-empty. If $\widetilde{\mathbf{P}}$ is empty, we choose $\mathbf{p}$ to be the infinity vector and say that the market breaks down and define all dealers' payoff to be zero. We refer to the collection of rules that define a unique vector $\mathbf{p}$ for any given realization of signals and strategy profile as $\mathbf{P}\left(\left\{\mathbf{Q}^i\left(s^i;\mathbf{p}_{g^i}\right)\right\}_i,\mathbf{s}\right)$. As in Vives (2011), we require that trades clear both on and off the equilibrium. Introducing the set (3) ensures that we can evaluate dealers' payoffs for any demand functions that dealers may choose. This will allow us to search for a Bayesian Nash equilibrium as explained in the following section.

2.2 Equilibrium concept

The environment described above represents a Bayesian game, henceforth the OTC game. The risk-neutrality of dealers and the normal information structure allows us to search for a linear equilibrium of this game defined as follows.

Definition 1 *A Linear Bayesian Nash equilibrium of the OTC game is a vector of linear generalized demand functions* $\left\{\mathbf{Q}^1(s^1;\mathbf{p}_{g^1}),\mathbf{Q}^2(s^2;\mathbf{p}_{g^2}),...,\mathbf{Q}^n(s^n;\mathbf{p}_{g^n})\right\}$ *such that* $\mathbf{Q}^i(s^i;\mathbf{p}_{g^i})$ *solves the problem*

$$\max_{(Q^i_{ij})_{j\in g^i}} E\left\{\left[\sum_{j\in g^i}Q^i_{ij}(s^i;\mathbf{p}_{g^i})\left(\theta^i-p_{ij}\right)\right]\middle|s^i,\mathbf{p}_{g^i}\right\}, \tag{4}$$

for each dealer i*, where* $\mathbf{p}=\mathbf{P}\left(\cdot,\mathbf{s}\right)$*.*

A dealer i chooses a demand function, $Q^i_{ij}\left(\cdot\right)$, for each transaction ij, in order to maximize her expected profits, given her information, s^i, and given the demand functions chosen by the other dealers. Implicit in the definition of the equilibrium is that each dealer understands she has a price impact when trading with the counterparties given by the network g. Solving problem (4) is equivalent to finding a fixed point in demand functions.

[5] The specific algorithm we choose to select a unique price vector is immaterial. To ensure that our game is well defined, we need to specify dealers' payoffs as they depend on their strategies both on and off the equilibrium path.

3 The Equilibrium

In this section, we derive the equilibrium in the OTC game. First, we derive the equilibrium strategies as a function of posterior beliefs. Second, we construct posterior beliefs. Third, we provide sufficient conditions for the existence of the equilibrium in the OTC game for any network.

3.1 Derivation of demand functions

Our derivation follows Kyle (1989) and Vives (2011). We conjecture an equilibrium in linear demand functions, such that the demand function of any given dealer i in the transaction with a counterparty j is

$$Q_{ij}^i(s^i; \mathbf{p}_{g^i}) = t_{ij}^i(y_{ij}^i s^i + \sum_{k \in g^i} z_{ij,ik}^i p_{ik} - p_{ij}). \tag{5}$$

We refer to t_{ij}^i as the *trading intensity* of dealer i on the link ij, while $z_{ij,ik}^i$ captures the effect specific to the price p_{ik} on the quantity that dealer i demands on the link ij. As it will become clear below, dealer i's best response is (5) when all other agents' demand functions are given by (5).

As is standard in similar models, we simplify the optimization problem (4), which is defined over a function space, to finding the functions $Q_{ij}^i(s^i; \mathbf{p}_{g^i})$ point-by-point. For this, we fix a realization of the vector of signals, $\mathbf{s}$. Then, we solve for the optimal quantity q_{ij}^i that each dealer i demands when trading with a counterparty j, as she takes as given the demand functions of the other dealers. Thus, we obtain dealer's i best response quantity q_{ij}^i in the transaction with dealer j, for each realization of the signals. This essentially gives us a map from prices to quantities, or her demand function. We describe the procedure in detail below.

Given the conjecture (5) and market clearing

$$Q_{ij}^i(s^i; \mathbf{p}_{g^i}) + Q_{ij}^j(s^j; \mathbf{p}_{g^j}) + \beta_{ij} p_{ij} = 0, \tag{6}$$

the residual inverse demand function of dealer i in a transaction with dealer j is

$$p_{ij} = -\frac{t_{ij}^j(y_{ij}^j s^j + \sum_{k \in g^j, k \neq i} z_{ij,jk}^j p_{jk}) + q_{ij}^i}{\beta_{ij} + t_{ij}^j \left(z_{ij,ij}^j - 1\right)}. \tag{7}$$

Denote

$$I_{ij}^{j} \equiv -\frac{t_{ij}^{j}(y_{ij}^{j}s^{j} + \sum_{k\in g^{j},k\neq i} z_{ij,jk}^{j} p_{jk})}{\beta_{ij} + t_{ij}^{i}\left(z_{ij,ij}^{j} - 1\right)} \tag{8}$$

and rewrite (7) as

$$p_{ij} = I_{ij}^{j} - \frac{1}{\beta_{ij} + t_{ij}^{j}\left(z_{ij,ij}^{j} - 1\right)} q_{ij}^{i}. \tag{9}$$

The uncertainty that dealer i faces about the signals of others is reflected in the random intercept of the residual inverse demand, I_{ij}^{j}, while her capacity to affect the price is reflected in the slope $-1/\left(\beta_{ij} + t_{ij}^{j}\left(z_{ij,ij}^{j} - 1\right)\right)$. Thus, the price p_{ij} is informationally equivalent to the intercept I_{ij}^{j}. This implies that finding the vector of quantities $\mathbf{q}^{i} = \mathbf{Q}^{i}(s^{i}; \mathbf{p}_{g^{i}})$ for one particular realization of the signals, $\mathbf{s}$, is equivalent to solving

$$\max_{(q_{ij}^{i})_{j\in g^{i}}} E\left[\sum_{j\in g^{i}} q_{ij}^{i}\left(\theta^{i} + \frac{1}{\beta_{ij} + t_{ij}^{j}\left(z_{ij,ij}^{j} - 1\right)} q_{ij}^{i} - I_{ij}^{j}\right) | s^{i}, \mathbf{p}_{g^{i}}\right],$$

or

$$\max_{(q_{ij}^{i})_{j\in g^{i}}} \sum_{j\in g^{i}} q_{ij}^{i}\left(E\left(\theta^{i}|s^{i}, \mathbf{p}_{g^{i}}\right) + \frac{1}{\beta_{ij} + t_{ij}^{j}\left(z_{ij,ij}^{j} - 1\right)} q_{ij}^{i} - I_{ij}^{j}\right).$$

From the first-order conditions, we derive the quantities q_{ij}^{i} for each link of i and for each realization of $\mathbf{s}$ as

$$2\frac{1}{\beta_{ij} + t_{ij}^{j}\left(z_{ij,ij}^{j} - 1\right)} q_{ij}^{i} = I_{ij}^{j} - E\left(\theta^{i}|s^{i}, \mathbf{p}_{g^{i}}\right).$$

Then, using (9), we can find the optimal demand function

$$Q_{ij}^{i}(s^{i}; \mathbf{p}_{g^{i}}) = -\left(\beta_{ij} + t_{ij}^{j}\left(z_{ij,ij}^{j} - 1\right)\right)\left(E(\theta^{i}\left|s^{i}, \mathbf{p}_{g^{i}}\right.) - p_{ij}\right) \tag{10}$$

for each dealer i when trading with dealer j.

Further, given our conjecture (5), equating coefficients in equation (10) implies that

$$E(\theta^{i}\left|s^{i}, \mathbf{p}_{g^{i}}\right.) = y_{ij}^{i}s^{i} + \sum_{k\in g^{i}} z_{ij,ik}^{i} p_{ik}.$$

However, the projection theorem implies that the belief of each dealer i can be described as a unique linear combination of her signal and the prices she observes. Thus, it must be that

$y^i_{ij} = y^i$, and $z^i_{ij,ik} = z^i_{ik}$ for all i, j, and k. In other words, the posterior belief of a dealer i is given by

$$E\left(\theta^i | s^i, \mathbf{p}_{g^i}\right) = y^i s^i + \mathbf{z}_{g^i} \mathbf{p}_{g^i}, \tag{11}$$

where $\mathbf{z}_{g^i} = \left(z^i_{ij}\right)_{j \in g^i}$ is a *row* vector of size m^i. Then, we obtain that the trading intensity of dealer i is the inverse of her price impact in the transaction with dealer j, or

$$t^i_{ij} = t^j_{ij}\left(1 - z^j_{ij}\right) - \beta_{ij}. \tag{12}$$

Substituting (11) back into our conjecture (5), we obtain that the demand of dealer i in a transaction with dealer j is given by

$$Q^i_{ij}(s^i; \mathbf{p}_{g^i}) = t^i_{ij}\left(E\left(\theta^i | s^i, \mathbf{p}_{g^i}\right) - p_{ij}\right). \tag{13}$$

That is, the quantity that dealer i trades with j is the perceived gain per unit of the asset, $\left(E\left(\theta^i | s^i, \mathbf{p}_{g^i}\right) - p_{ij}\right)$, multiplied by the endogenous trading intensity parameter, t^i_{ij}. Moreover, by substituting the optimal demand function (13) into the bilateral market clearing condition (6), we obtain the equilibrium price between any pair of dealers i and j as a linear combination of the posterior beliefs of i and j:

$$p_{ij} = \frac{t^i_{ij} E(\theta^i \,| s^i, \mathbf{p}_{g^i}) + t^j_{ij} E(\theta^j \,| s^i, \mathbf{p}_{g^j})}{t^i_{ij} + t^j_{ij} - \beta_{ij}}, \tag{14}$$

At this point we depart from the standard derivation. The standard approach is to determine the coefficients of the demand function (5) using a fixed-point argument. In particular, given our conjecture (5), the bilateral clearing conditions represent a system of linear equations from which prices can be derived as an affine combination of signals. Then, the projection theorem implies that for each dealer i, the coefficients y^i and $\mathbf{z}_{g^i}$ must satisfy the following fixed-point condition

$$\begin{bmatrix} y^i \\ \mathbf{z}^\top_{g^i} \end{bmatrix} = \mathcal{V}\left(\theta^i, \begin{bmatrix} s^i \\ \mathbf{p}_{g^i} \end{bmatrix}\right) \times \left(\mathcal{V}\begin{bmatrix} s^i \\ \mathbf{p}_{g^i} \end{bmatrix}\right)^{-1}. \tag{15}$$

Note that if (15) has a solution for each dealer i, equation (10) implies that our conjecture (5) verifies.

In general networks, this procedure yields a high dimensional problem. First, the system of bilateral clearing conditions (6) has as many equations as the number of links in the network. Second, for each dealer we need to solve a fixed-point problem that is itself a function of her position in the network.

Our main methodological innovation is that we pin down the equilibrium of the OTC game in two steps. First, we construct the equilibrium posterior beliefs without solving for the demand curve or the implied quantities and prices. For this, we introduce in Section 3.2 an auxiliary game called the *conditional guessing game*.

Second, based on the equilibrium beliefs in the conditional guessing game, we construct the equilibrium demand functions of the OTC game in Section 3.3. We provide conditions for the existence of an equilibrium. In Section 4, we also formally state and qualify the one-to-one mapping of the posterior beliefs in the two games.

3.2 Deriving posterior beliefs: The conditional guessing game

We define the conditional guessing game as follows. Consider a set of n agents that are connected in the same network g as in the corresponding OTC game. The information structure is also the same as in the OTC game. Before the uncertainty is resolved, each agent i makes a guess, e^i, about her value of the asset, θ^i. Her guess is the outcome of a function that has as arguments the guesses of other dealers she is connected to in the network g. In particular, given her signal, dealer i chooses a guess function, $\mathcal{E}^i\left(s^i;\mathbf{e}_{g^i}\right)$, that maps the vector of guesses of her neighbors, $\mathbf{e}_{g^i}$, into a guess e^i. When the uncertainty is resolved, agent i receives a payoff $-\left(\theta^i-e^i\right)^2$, where e^i is an element of the guess vector $\mathbf{e}$ defined by the smallest element of the set

$$\Xi\left(\left\{\mathcal{E}^i\left(s^i;\mathbf{e}_{g^i}\right)\right\}^i,\mathbf{s}\right)\equiv\left\{\mathbf{e}\,\middle|\,e^i=\mathcal{E}^i\left(s^i;\mathbf{e}_{g^i}\right),\,\forall\ i\right\},\tag{16}$$

by lexicographical ordering. We assume that if a fixed-point in (16) did not exist, then dealers would not make any guesses and their payoffs would be set to minus infinity. Essentially, the set of conditions (16) is the counterpart in the conditional guessing game of the market clearing conditions in the OTC game.

Definition 2 *An equilibrium of the conditional guessing game is given by a strategy profile*

$\left(\mathcal{E}^1, \mathcal{E}^2, ..., \mathcal{E}^n\right)$ *such that each agent i chooses strategy $\mathcal{E}^i : R \times R^{m^i} \rightarrow R$ in order to maximize her expected payoff*

$$\max_{\mathcal{E}^i} \left\{ -E\left(\left(\theta^i - \mathcal{E}^i\left(s^i; \mathbf{e}_{g^i}\right)\right)^2 | s^i, \mathbf{e}_{g^i} \right) \right\},$$

where $\mathbf{e} = \Xi(\cdot, \mathbf{s})$.

As in the OTC game, we simplify this optimization problem and find the guess functions $\mathcal{E}^i\left(s^i; \mathbf{e}_{g^i}\right)$ point-by-point. That is, for each realization of the signals, $\mathbf{s}$, an agent i chooses a guess that maximizes her expected profits, given her information, s^i, and given the guess functions chosen by the other agents. Her optimal guess function is then given by

$$\mathcal{E}^i\left(s^i; \mathbf{e}_{g^i}\right) = E\left(\theta^i | s^i, \mathbf{e}_{g^i}\right). \tag{17}$$

In the next proposition, we state that the guessing game has an equilibrium in any network.

Proposition 1 *In the conditional guessing game, for any network g, there exists an equilibrium in linear guess functions, such that*

$$\mathcal{E}^i\left(s^i; \mathbf{e}_{g^i}\right) = \bar{y}^i s^i + \bar{\mathbf{z}}_{g^i}\mathbf{e}_{g^i}$$

for any i, where $\bar{y}^i$ is a scalar and $\bar{\mathbf{z}}_{g^i} = \left(\bar{z}^i_{ij}\right)_{j \in g^i}$ is a row vector of length m^i.

We derive the equilibrium in the conditional guessing game as a fixed point problem in the space of $n \times n$ matrices. In particular, consider an arbitrary $n \times n$ matrix $\overset{\prime}{V} = \left[\overset{\prime}{\mathbf{v}}{}^i\right]_{i=1,..n}$ and let the guess of each agent i be

$$\overset{\prime}{e}{}^i = \overset{\prime}{\mathbf{v}}{}^i \mathbf{s}, \tag{18}$$

given a realization of the signals $\mathbf{s}$. It follows that, when dealer j takes as given the choices of her neighbors, $\overset{\prime}{\mathbf{e}}_{g^j}$, her best response guess is

$$\overset{\prime\prime}{e}{}^j = E\left(\theta^j | s^j, \overset{\prime}{\mathbf{e}}_{g^j}\right). \tag{19}$$

Since each element of $\overset{\prime}{\mathbf{e}}_{g^j}$ is a linear function of the signals and the conditional expectation is a linear operator for jointly normally distributed variables, equation (19) implies that there is a unique vector $\overset{''}{\mathbf{v}}^j$, such that

$$\overset{''}{e}^j = \overset{''}{\mathbf{v}}^j \mathbf{s}. \tag{20}$$

In other words, the conditional expectation operator defines a mapping from the $n \times n$ matrix $\overset{'}{V} = \left[\overset{'}{\mathbf{v}}^i\right]_{i=1,..n}$ to a new matrix of the same size $\overset{''}{V} = \left[\overset{''}{\mathbf{v}}^i\right]_{i=1,..n}$. An equilibrium of the conditional guessing game exists if this mapping has a fixed point. Proposition 1 shows the existence of a fixed point and describes the equilibrium as given by the coefficients of s^i and $\mathbf{e}_{g^i}$ in $E\left(\theta^i | s^i, \mathbf{e}_{g^i}\right)$ at this fixed point.

Next, we use the conditional guessing game to establish conditions for the existence of an equilibrium in the OTC game, we show how to solve for the equilibrium coefficients. In the following section, we also prove that posterior beliefs of the OTC game coincide with the equilibrium beliefs in the conditional guessing game.

3.3 Solving for equilibrium coefficients and existence

In this part, we prove the main results of this section. In particular, we provide sufficient conditions under which we can construct an equilibrium of the OTC game building on an equilibrium of the conditional guessing game.

Proposition 2 *Let $\bar{y}^i$ and $\bar{\mathbf{z}}_{g^i} = \left(\bar{z}^i_{ij}\right)_{j \in g^i}$ be the coefficients that support an equilibrium in the conditional guessing game and let $e^i = E(\theta^i \left| s^i, \mathbf{e}_{g^i}\right.)$ be the corresponding equilibrium expectation of agent i. Then, there exists a Linear Bayesian Nash equilibrium in the OTC game, whenever $\rho < 1$ and the following system*

$$\begin{aligned} \frac{y^i}{\left(1 - \sum\limits_{k \in g^i} z^i_{ik} \frac{2 - z^k_{ki}}{4 - z^i_{ik} z^k_{ki}}\right)} &= \bar{y}^i \\ z^i_{ij} \frac{\frac{2 - z^i_{ij}}{4 - z^i_{ij} z^j_{ji}}}{\left(1 - \sum\limits_{k \in g^i} z^i_{ik} \frac{2 - z^k_{ki}}{4 - z^i_{ik} z^k_{ki}}\right)} &= \bar{z}^i_{ij}, \forall j \in g^i \end{aligned} \tag{21}$$

has a solution $\left\{y^i, z^i_{ij}\right\}_{i=1,..n, j \in g^i}$ such that $z^i_{ij} \in (0, 2)$. The equilibrium demand functions are *given by (5) with*

$$t_{ij}^i = -\beta_{ij} \frac{2 - z_{ij}^j}{z_{ij}^i + z_{ji}^j - z_{ij}^i z_{ji}^j}. \tag{22}$$

The equilibrium beliefs are $E\left(\theta^i | s^i, \mathbf{p}_{g^i}\right) = y^i s^i + \sum_{j \in g^i} z_{ij}^i p_{ij}$, *while the equilibrium prices and quantities are*

$$p_{ij} = \frac{t_{ij}^i e^i + t_{ij}^j e^j}{t_{ij}^i + t_{ij}^j - \beta_{ij}} \tag{23}$$

$$q_{ij}^i = t_{ij}^i \left(e^i - p_{ij}\right). \tag{24}$$

The conceptual advantage of our way of constructing the equilibrium over the standard approach is that it is based on a simpler fixed-point problem. Indeed, in the conditional guessing game we solve for a fixed point in beliefs. This simplifies the fixed point problem as there is only n guessing functions as opposed to $\left(\Sigma_i m^i\right)$ demand functions. Then, the system of equations (21) ensures we can map n expectations, e^i, from the conditional guessing game into $M \geq n$ prices in the OTC game, in a consistent way.

Note also that Proposition 2 also describes a simple numerical algorithm to find the equilibrium of the OTC game for any network. In particular, the conditional guessing game gives parameters $\bar{y}^i$ and $\bar{z}_{ij}$, and conditions (21) imply parameters y^i and z_{ij}. Making use of (22), we then obtain the demand functions that imply prices and quantities by (23)-(24).

The next proposition strengthens the existence result for our specific examples.

Proposition 3

1. *In any network in the circulant family, the equilibrium of the OTC game exists.*

2. *In a star network, the equilibrium of the OTC game exists.*

For the star network and the complete network, closed-form solutions are derived in Appendix B.

We showed in Proposition 2 that an equilibrium exists when the solution, z_{ij}^i, of the system (21) is in the interval $(0, 2)$. As Section 6 illustrates, apart from the networks characterized in Proposition 3, we found that the equilibrium exists for a large range of parameters for a wide range of relevant random networks. However, there exist parameters for which in certain irregular networks the conditions of Proposition 2 are not satisfied. We report and explain the simplest such example in Appendix F.[6]

We conclude this section with the observation that costumers' demand plays a limited role in our analysis. While there is no equilibrium for $\beta_{ij} = 0$, for any choice of $\beta_{ij} < 0$, prices, beliefs and scaled quantities $\frac{q^i_{ij}}{\beta_{ij}}$ are not affected. We summarize this in the following Corollary.

Corollary 1 *Prices, beliefs and scaled quantities, $\frac{q^i_{ij}}{\beta_{ij}}$, are independent of the slope of customers' demand, β_{ij}. Furthermore, if $\beta_{ij} = \hat{\beta} \cdot \hat{\beta}_{ij}$, where each $\hat{\beta}_{ij}$ is an arbitrary negative scalar and $\hat{\beta}$ is a positive constant, then prices, beliefs and scaled quantities remain constant and well defined as $\hat{\beta} \to 0$. When $\hat{\beta} = 0$, the equilibrium in the OTC game does not exist.*

This result follows immediately from (22) and (24). Clearly, beliefs must be independent of customers' demand as they can be derived from the conditional guessing game where there are no customers. Quantities, q^i_{ij}, are proportional to β_{ij}, because trading intensities, t^i_{ij}, are. This is immediate from the fact that β_{ij} is a parallel shift in expression (12), which drives the equilibrium trading intensities.

Intuitively, we need a non-zero β_{ij} as $\frac{1}{\beta_{ij}}$ serves as a finite upper bound for the price impact of an additional unit supplied in a transaction between i and j. This is apparent from (9). To see why this is essential, it is useful to think about equation (26) as a best response function for trading intensities. If β_{ij} were 0, then counterparties' best responses would converge to zero as $|(1 - z_{ij})| < 1$ by the conditions required in Proposition 2. That is, trade would collapse. This is a well known property of similar games (e.g., Kyle (1989) for the case of two agents). Based on Corollary 1, we argue that the exogenous demand from customers solves this technical problem with minimal impact on the results.

4 Information Diffusion

In this section, we discuss informational properties of prices in the OTC market. First, we characterize the role of the market structure in the diffusion of information through prices.

[6] In these cases, there is at least one agent who puts negative weight on at least one of her neighbours' expectations, that is, $\bar{z}^i_{ij} < 0$ for some i and ij. This is possible as the correlation between θ^i and e^j, conditional on all the other expectations of i's neighbours and s^i might be negative. While this is still a valid equilbrium of the conditional guessing game, it results in a negative z^i_{ij} in the OTC game, which violates the second-order conditons.

Second, we introduce a measure of informational efficiency and highlight inefficiencies in how agents learn from prices.

4.1 Prices and Information Diffusion

We study how the market structure affects the diffusion of information through the network or trades. For this, we analyze two dimensions. First, we are interested in finding out to what extent the ability of agents to behave strategically and impact prices influences how much information gets revealed. Second, we investigate how the network structure interacts with the role of prices as information aggregators.

To evaluate the role of agents' strategic motives when trading, the conditional guessing game is a useful benchmark. This is because any considerations related to price manipulation are not present in the conditional guessing game. We establish the following result.

Proposition 4 *In any Linear Bayesian Nash equilibrium of the OTC game the vector with elements e^i defined as*

$$e^i = E(\theta^i \left| s^i, \mathbf{p}_{g^i} \right.)$$

is an equilibrium expectation vector in the conditional guessing game.

The idea behind this proposition is as follows. We have already shown that in a linear equilibrium, each bilateral price p_{ij} is a linear combination of the posteriors of i and j, $E(\theta^i \left| s^i, \mathbf{p}_{g^i} \right.)$ and $E(\theta^j \left| s^j, \mathbf{p}_{g^j} \right.)$, as described in (14). Therefore, in each transaction, given that a dealer knows her own belief, the price reveals the belief of her counterparty. Thus, when a dealer chooses her generalized demand function, she essentially conditions her expectation about the asset value on the expectations of the other dealers she is trading with. Consequently, the set of posteriors implied in the OTC game works also as an equilibrium in the conditional guessing game.

The equivalence of beliefs on the two games implies that any feature of the beliefs in the OTC game must be unrelated in any way to price manipulation, market power or other profit related motives.

Next, we analyze the role of the network structure in how prices aggregate information. We obtain the following result for general connected networks.

Proposition 5 *Suppose that there exists an equilibrium in the OTC game. Then in any connected network g, each bilateral price is a linear combination of all signals in the economy, with strictly positive weight on each signal.*

This result suggests that a decentralized trading structure can be surprisingly effective in transmitting information. Indeed, although we consider only a single round of transactions, each price partially incorporates all the private signals in the economy. A simple way to see this is to consider the residual demand curve and its intercept, I_{ij}^{i}, defined in (8)-(9). This intercept is stochastic and informationally equivalent with the price p_{ij}. The chain structure embedded in the definition of I_{ij}^{i} is critical. The price p_{ij} gives information on I_{i}^{j}, which gives some information on the prices agent j trades at in equilibrium. For example, if agent j trades with agent k then p_{jk} affects p_{ij}. By the same logic, p_{jk} in turn is affected by the prices agent k trades at with her counterparties, etc. Therefore, p_{ij} aggregates the private information of signals of every agent, dealer i is indirectly connected to, even if this connection is through several intermediaries.

Typically, however, dealers in the OTC market do not learn from prices all the relevant information in the economy. This is because in a network g, a dealer i can use only m^i linear combinations of the vector of signals, $\mathbf{s}$, to infer the informational content of the other $(n-1)$ signals. In contrast, as Vives (2011) shows, in a centralized market in which each agent chooses one demand function and the market clears at a single price, a dealer i learns all the relevant information in the economy, and her posterior belief is given by $E\left(\theta^i|\mathbf{s}\right)$.

There are two special cases when the prices are privately fully revealing if agents trade over the counter. In our context, the equilibrium prices are *privately fully revealing* if for each dealer i, $(s^i, \mathbf{p}_{g^i})$ is a sufficient statistic of the vector of signals $\mathbf{s}$, in the estimation of θ^i. The following result describes these cases.

Proposition 6

1. *In the complete network, prices are privately fully revealing.*

2. *In any connected network, g, prices are privately fully revealing when $\rho \rightarrow 1$.*

The first case follows immediately. In a complete network, each agent has $m^i = n - 1$

neighbors, thus she observes $n-1$ prices. Given that she know her own signal, she can in equilibrium invert the prices to obtain the signals of the other dealers.

The second case in Proposition 6 shows that in the common value limit, the network structure does not impose any friction on the information transmission process in any network. To shed more light on the intuition behind the latter result, we appeal to Proposition 4 and build intuition based on the learning process in the conditional guessing game.

Consider the case when $\rho \to 1$. As the private value component in θ^i vanishes, if any dealer could observe all the signals, their best guess would be $E\left(\theta^i|\mathbf{s}\right) = \frac{\sigma_\theta^2}{n\sigma_\theta^2+\sigma_\varepsilon^2}\mathbf{1}^\top\mathbf{s}$. Therefore, in the conditional game, if dealer i could learn the sum of signals, $\mathbf{1}^\top\mathbf{s}$, from her neighbors' guesses, $\mathbf{e}_{g^i}$, her guess would converge to $e^i = \frac{\sigma_\theta^2}{n\sigma_\theta^2+\sigma_\varepsilon^2}\mathbf{1}^\top\mathbf{s}$. At the same time, dealer's i guess reveals $\mathbf{1}^\top\mathbf{s}$ to all her neighbors. That is, the guess $e^i = \frac{\sigma_\theta^2}{n\sigma_\theta^2+\sigma_\varepsilon^2}\mathbf{1}^\top\mathbf{s}$ is a fixed point of the system (18)-(20). Hence, this is an equilibrium of the conditional guessing game. Since the equilibrium is continuos in ρ, by Proposition 4, information is fully revealed in the OTC game as well when $\rho \to 1$. Nevertheless, following the argument in Vives (2011), there is no equilibrium of the OTC game exactly at $\rho = 1$. The reason is that if the price reveals the common value, then no dealer has an incentive to put any weight on her signal. A paradox arises as then the price cannot contain any information.

4.2 Informational Efficiency

In this section we discuss the informational efficiency of prices. We defer the discussion on allocative inefficiency to Section 5.

As we have seen above, information is generally not fully revealed in the equilibrium of the OTC trading game, apart from the two cases discussed in Proposition 6. Moreover, no single price fully reveals all the information, except in the common value limit. Thus, we propose a measure of informational efficiency based on dealers' beliefs, taking into account that their learning is constrained by the network structure. More precisely, we exploit the equivalence of beliefs in Proposition 4 and define a measure of constrained informational efficiency as the negative sum of squared deviations from the true value,

$$U\left(\left\{\bar{y}^i, \bar{\mathbf{z}}_{g^i}\right\}_{i\in\{1,\ldots n\}}\right) \equiv -E\left[\sum_i \left(\theta^i - \mathcal{E}^i\left(s^i;\mathbf{e}_{g^i}\right)\right)^2 \middle| \mathbf{s}\right], \tag{25}$$

where $\mathcal{E}^i(\cdot)$ is the guess function of a dealer i in the conditional guessing game. Then, we can find conditional guessing functions $\left\{\mathcal{E}^i\left(s^i;\mathbf{e}_{g^i}\right)\right\}_{i=1\ldots n}$ which would maximize our measure of constrained informational efficiency (25) subject to $\mathbf{e}=\Xi(\cdot,\mathbf{s})$ and (16). This is the planner's solution in the conditional guessing game. Alternatively, we can also look for marginal deviations in dealer's equilibrium strategies in the conditional guessing game (which, by Proposition 2, would correspond to marginal deviations from equilibrium strategies in the OTC game) which would improve constrained informational efficiency.

In general, we find that beliefs are not constrained informationally efficient. We illustrate the underlying informational externality on the circle and star networks in this section, and show that this observation is robust to a large set of random networks in Section 6.

Since in a circle all dealers are symmetric, and each can learn only from two prices, this is the simplest example to recover the learning externality that leads to informational inefficiencies. To see the intuition, we use expressions (18)-(20) as an iterated algorithm of best responses. That is, in the first round, each agent i receives an initial vector of guesses, $\mathbf{e}'_{g^i}$, from her neighbors. Given this, each agent i chooses her best guess, $e^{\prime\prime i}$, as in (19). The vector of guesses $\mathbf{e}''_{g^i}$, with elements given by (20), is the starting point for i in the following round. By definition, if the algorithm converges to a fixed point, then this is an equilibrium of the conditional guessing game.

We chose an example with eleven dealers to have a sufficient number of iterations. We illustrate the iteration rounds in Figure 1 from the point of view of dealer 6. We plot the weights with which signals are incorporated in the guess of dealer 5, 6 and 7, i.e. $\mathbf{v}^5,\mathbf{v}^6,\mathbf{v}^7$. In each figure, the dashed lines show the posteriors of dealer 5 and 7 at the beginning of each round, and the solid line shows the posterior of agent 6 at the end of each round after she observes her neighbors' guesses. We start the algorithm by assuming that the posteriors of dealer 5 and 7 are the posteriors in the common value limit, $\frac{\sigma_\theta^2}{n\sigma_\theta^2+\sigma_\varepsilon^2}\mathbf{1}^\top s$, as illustrated by the straight dashed lines that overlap in panel A. The best response guess of dealer 6 at the end of round 1 is shown by the solid line peaking at s^6 in Panel A. The reason dealer 6 puts more weight on her signal, s^6, is that it is more informative about her value, θ^6, than the rest of the signals. Clearly, this is not a fixed point as all other agents choose their guesses in the same way. Thus, in round 2, agent 6 observes posteriors that are represented by the dashed lines shown on Panel B; these are the mirror images of the round-1 guess of dealer 6. Note that the posteriors that dealers 5 and 7 hold at the beginning of round 2 are less informative for dealer 6 than the equal-weighted sum of signals $\frac{\sigma_\theta^2}{n\sigma_\theta^2+\sigma_\varepsilon^2}\mathbf{1}^\top s$. The reason is that, for dealer 6, her signal together with the equal-weighted sum of signals is a sufficient statistic for all the information in the economy. Thus, while in round 1 she learned everything she wanted to learn, in round 2 she cannot. The weight that dealer 5 and 7 place on their own private signals "jams" the information content of the guesses that dealer 6 observes. Nevertheless, the round-2 guesses are informative, and dealer 6 updates her posterior by placing a larger weight on her own signal, as the solid line on Panel B shows. Since all other agents update their posterior in a similar way, the guesses that dealer 6 observes in round 3 are a mirror image of her own guess, as shown by the dashed lines in Panel C. The solid line in Panel C represents dealer 6's guess in round 4. On Panel D, we depict the guess of dealer 6 in each round until round 5, where we reach the fixed point.

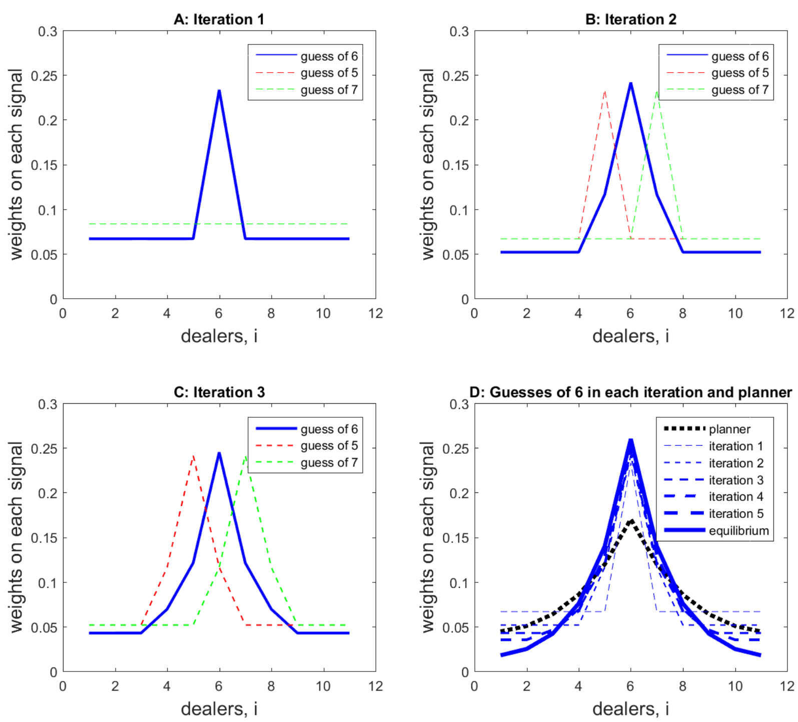

Figure 1: Best responses in the conditional guessing game in a ring network. Panel A shows of player 6's best response weight on each signal when her neihbours' guess weighs each signal uniformly at $\frac{\sigma_\theta^2}{n\sigma_\theta^2+\sigma_\varepsilon^2}\mathbf{1}^\top$. Panel B-D shows further iterations of best responses. Panel D also shows the planner's solution. Parameters are $n = 11, \rho = 0.8, \sigma_\theta^2 = \sigma_\varepsilon^2 = 1, \beta_{ij} = -1$.

The thick dashed curve in the last panel of Figure 1 shows the optimal weights on each signal in the belief of dealer 6 in the planner's solution. As is apparent, the dealer places more weight on her own signal in equilibrium than what is informationally efficient. The reason is that each agent's conditional guess function affects how much her neighbors can learn from her guess. This, in turn, affects the learning of her neighbors' neighbors, etc. Although dealers optimally choose guesses that are tilted towards their own signals, they do not internalize that they distort the informational content of these guesses for others.

In the following proposition, we show that this observation is not unique to the example. Indeed, in any star network, the sum of payoffs would increase if, starting from the decentralized equilibrium, both central and periphery dealers would put less weight on their respective signal and more weight on their neighbors' guesses.

Proposition 7 *Let* $U\left(\left\{\bar{y}^i, \bar{\mathbf{z}}_{g^i}\right\}_{i\in\{1,\ldots n\}}\right)$ *be the sum of payoffs in an star network for any given strategy profile* $\left\{\bar{y}^i, \bar{\mathbf{z}}_{g^i}\right\}_{i\in\{1,\ldots n\}}$. *Then, if* $\left\{\bar{y}^{i*}, \bar{\mathbf{z}}^*_{g^i}\right\}_{i\in\{1,\ldots n\}}$ *is the decentralized equilibrium, then*

$$\lim_{\delta\to 0}\frac{\partial U\left(\left\{\bar{y}^{i*}-\delta, \bar{\mathbf{z}}^*_{g^i}+\delta\mathbf{1}\right\}_{i\in\{1,\ldots n\}}\right)}{\partial\delta} > 0.$$

That is, starting from the equilibrium solution, marginally decreasing weights on a dealer's signal, while marginally increasing weights on other dealers' guesses increases the sum of payoffs.

The intuition we provide about why dealers overweight their signal in a circle network is informative as well about why the central dealers overweight their signal in a star network. The planner would prefer the central dealer to put less weight on her own signal, as this would make her guess more informative on the common value component, that is, more useful for the periphery agents. In turn, once the guess of the central agent is more informative, the periphery agents should put more weight on that and less on their own signal. This explains why periphery agents overweight their signal in the decentralized solution.

Note that this informational inefficiency does not arise as a result of imperfect competition, or strategic trading motives that agents have. Indeed, the equivalence between dealers' beliefs in the conditional guessing game and in the OTC game implies that this is not the case. Instead, it is a consequence of the learning externality arising from the interaction between the interdependent value environment and the network structure.

An interesting question is whether the informational inefficiency can be corrected to some degree. It is a reasonable conjecture that when signals are costly to acquire, dealers may put less weight on their signal relative to the information they learn from prices than when the signals are costless. However, how dealers would best respond to each others' choices of information precision, how the properties of the remaining equilibrium would change with the network structure, and how it would compare to the planner's solution are non-trivial questions which we leave for future research.

5 Profit, welfare, and illiquidity

In this section, we provide insights on how the OTC market structure and adverse selection affect dealers expected profit, welfare, and illiquidity. First, we make general observations about the intuition driving these objects in any network. Then we proceed to give further insights by analyzing the most common OTC networks. In particular, we isolate the effect of decentralization by comparing the complete OTC network with centralized markets; we illustrate the role of link density by comparing circulant OTC networks in which we successively increase the number of links each dealer has; and we analyze the effect of asymmetric number of links in the star OTC network.

To keep the market structures comparable, we assume that dealers have an identically sized customer pool. To simplify the welfare analysis, we assume that dealers charge a zero mark-up. As before, a dealer i in the OTC market uses each link ij to satisfy an exogenous fraction of her customer base. This implies that in the centralized market, the absolute slope of the customers demand is $-\beta_V = nB$, while in any OTC markets with K total links the customers' demand in any transaction between dealer i and j is $-\beta_{ij} = \frac{nB}{K}$, where $B > 0$ is an exogenous constant.

Section 6 complements this analysis by checking the robustness of our insights on realistic random networks.

5.1 General observations

Before the formal analysis, it is instructive to explain the intuition on what might determine traders' profit and total welfare in our economy. First of all, recall that each dealer is risk neutral and their valuation has a private component. This implies that if all dealers would take unboundedly large negative or positive positions, that could lead to unboundedly large expected profit and welfare. As an illustration, consider the following (non-equilibrium) allocation. Let the posterior expectations e^i be determined in the equilibrium of the conditional guessing game and let prices and traded quantities be fixed at

$$p_{ij} = \frac{e^i + e^j}{2}, \; q_{ij}^i = t\left(e^i - p_{ij}\right),$$

where the trading intensity, t, is the same arbitrary positive constant for each agent. It is easy to check that as each dealer trades in the direction of her posterior, increasing t without bound would increase expected profit and total welfare without bound.

In equilibrium, dealers do not take infinite positions as they are concerned about adverse selection. While expressions (23) and (24) for prices and quantities are similar to the thought experiment above, the trading intensity of each dealer, t_{ij}^i, is determined as in equilibrium from the best response function given by expressions (11) and

$$t_{ij}^i = t_{ij}^j\left(1 - z_{ij}^j\right) - \beta_{ij}. \tag{26}$$

Since the coefficients of prices in posteriors, z_{ij}^j, depend on the network structure, the trading intensities depend as well. By solving for the trading intensities while keeping z_{ij}^j and z_{ij}^i constant, we obtain the equilibrium expression (22). Note that this expression implies $\frac{\partial t_{ij}^i}{\partial z_{ij}^j} < 0$. That is, the trading intensity of dealer i is smaller if her counterparty puts a larger weight on the price p_{ij} when forming her expectation. We should expect z_{ij}^j to be higher when the price p_{ij} is a more important source of information for j because either i observes more prices, j observes fewer prices, or the correlation across values is small. Therefore, z_{ij}^j is a natural measure of how much dealer j is concerned about adverse selection when trading with dealer i.

As $\frac{1}{t_{ij}^i}$ is the price impact of a unit of trade of i at link ij, another way to rephrase our observations is that the more j is concerned about adverse selection, the less liquid the trade is for dealer i. Hence, she trades with a lower trading intensity. Averaging $\frac{1}{t_{ij}^i}$ over the links of i provides a natural, dealer-level illiquidity measure we use to compare illiquidity across market structures from i's perspective. We use illiquidity, cost of trading, and price impact interchangeably.

We naturally expect the average profit of dealer i

$$E\left(\sum_{ij\in g^i} q_{ij}^i\left(\theta^i - p_{ij}\right)\right) = \sum_{ij\in g^i} t_{ij}^i E\left(\left(e^i - p_{ij}\right)^2\right), \tag{27}$$

to increase with the number of links as this implies both more opportunities to trade and intermediate, as well as larger trading intensities. While the expected profit also depends on the gains per unit of trade, $E\left(\left(e^i - p_{ij}\right)^2\right)$ at each link, we find in all our examples that variation in trading intensities and the opportunities for intermediation are the driving forces.

To compare welfare implied by different dealer networks, we also need the expected utility of customers. Customers' expected utility at link ij is proportional to the variance of the price p_{ij} since

$$E\left(\frac{\left(-\left(q_{ij}^j + q_{ij}^i\right)\right)^2}{2\beta_{ij}} + \left(q_{ij}^i + q_{ij}^j\right)p_{ij}\right) = \frac{\beta_{ij}}{2}E\left(p_{ij}^2\right) - \beta_{ij}E\left(p_{ij}^2\right) = -\frac{\beta_{ij}}{2}E\left(p_{ij}^2\right), \tag{28}$$

as it follows from market clearing. Hence, total welfare is the sum of profits and customers' utility summed over each link of the network

$$\sum_{ij\in g}\left(-\frac{\beta_{ij}}{2}E\left(p_{ij}\right)^2 + E\left(q_{ij}^i\left(\theta^i - p_{ij}\right)\right) + E\left(q_{ij}^j\left(\theta^j - p_{ij}\right)\right)\right). \tag{29}$$

Finally, note that from (23) it is immediate that price dispersion arises naturally in this model. A dealer with multiple trading partners is trading the same asset at various prices, because she is facing different demand curves along each link. Just as a monopolist does in a standard price-discrimination setting, this dealer sets a higher price in the market where demand is higher. In fact, from (23), we can foresee that the price dispersion in our framework must be closely related to the dispersion of posterior beliefs.

5.2 The effect of decentralized trading: the centralized and the complete network OTC market

Comparing the equilibrium in a centralized market as described in Vives (2011) with the equilibrium in the OTC complete network isolates the effect of trade decentralization. While in both cases posterior expectations are the same (and efficiently incorporate all the information in the market) and each trader can trade with all the others, still, prices, allocations, and welfare differ. The main observation in this part is that the effect of trade decentralization on welfare and illiquidity is ambiguous. Close to the common value limit, the OTC market is more liquid and provides higher total welfare than the centralized market, while for lower correlations across values typically the opposite is true.

In Appendix B.1 we report closed-form solutions for the price, p_V, quantity, q_V, and the price coefficient in expectations, z_V, for centralized markets (i.e., Vives (2011)). The trading intensity of a dealer in a centralized market is given by $t_V = \frac{-\beta_V}{n(z_V-1)+2-z_V}$, which is the fixed point of expression

$$t^i = (n-1)\, t^{-i} \left(1 - z_V^{-i}\right) - \beta_V. \tag{30}$$

Equation 30 shows how the trading intensity, t^i, of dealer i responds to the trading intensity of all other agents, t^{-i}, and to their adverse selection concern, z_V^{-i}. This is the centralized counterpart of (26). Expected profit and welfare are calculated by trivial modifications of (27) and (29).

Importantly, Vives (2011) shows that there is linear equilibrium in centralized markets, if and only if $1 - \frac{1}{n-1} < z_V$. Just as we argued above, with risk-neutral dealers adverse selection concerns determine the slope of demand curves. It turns out that in a centralized market this concern has to be sufficiently strong, otherwise an equilibrium with finite slopes cannot be sustained. (The same condition is also required in for an equilibrium to exist in an OTC market. However, with bilateral trades it reduces to $0 < z_{ij}^j$.)

In a complete network, trading intensity is $t_{ij}^j = t_{ij}^i = t_{CN} = -\beta_{CN} \frac{1}{z_{CN}}$ and a closed-form for the adverse selection parameter z_{CN} is given in Appendix B.3. Also, as we explained, we keep the total mass of customers constant across the two market structures, implying $-\beta_{CN} = \frac{2B}{n-1}$ and $-\beta_V = Bn$ for some $B > 0$.

Panels A-D in Figure 2 illustrate how dealers' profit, customers' utility, illiquidity and total welfare compare across the two markets for different values of ρ, fixing all the other parameters. Also, in the next proposition, we state the general claims corresponding to these figures. Then, we discuss the mechanisms behind the results.

Proposition 8 *Comparing a centralized market with a complete network OTC market*

1. *When ρ or $\frac{\sigma_\varepsilon^2}{\sigma_\theta^2}$ is sufficiently low, such that z_V converges to $1-\frac{1}{1-n}$ from above, total welfare, and dealers' profits are larger and illiquidity is smaller in the centralized market.*

2. *When ρ is sufficiently close to 1, then*

   (a) *total welfare and customers' utility are higher and illiquidity is lower in the OTC market, while*

   (b) *dealers' profits are higher in the centralized market.*

The intuition is as follows. Observe first that as $z_V \to 1-\frac{1}{n-1}$ from above, trading intensity grows without bound, $t_V \to \infty$, and illiquidity falls to zero. As in the centralized market, the information content of the price is increasing with ρ and $\frac{\sigma_\varepsilon^2}{\sigma_\theta^2}$, and so does z_V. This implies that for sufficiently low ρ and $\frac{\sigma_\varepsilon^2}{\sigma_\theta^2}$, welfare and dealers' profit are increasing without bound in a centralized market. This is so, because as adverse selection gets weaker, dealers are ready to take on very large bets. Because the private value component implies gains from trade, these large trades translate into high expected profit and high welfare. Given that quantities and profits are finite in the OTC market as long as ρ is not close to 0, it immediately follows that at least when z_V is close to $1-\frac{1}{n-1}$, profit and welfare are larger, and illiquidity is lower in centralized markets.

Perhaps more surprising is that in the common value limit, when ρ is close to 1, total welfare is higher and illiquidity is lower in the OTC market than in the centralized market.

We start with the result on illiquidity. There are two forces that drive this result. First, even if the parameter z were the same under the two market structures, mechanical differences in best responses in (30) and (26) would lead to a different outcome. Namely, the absolute values of both the slope and and the intercept of best responses are higher in the centralized market. The slope is higher because the aggregate response of $(n-1)$ counterparties is larger than that of a single counterparty, while the intercept is higher because all customers are present in the centralized market: $-\beta_V = Bn > \frac{2B}{n-1} = -\beta_{CN}$. While slope and intercept have opposite effects, simple algebra shows that the sum of these forces would result in higher illiquidity in the OTC market as $\frac{\frac{1}{t_V}}{\frac{1}{t_{CN}}}|_{z_v=z_{CN}=z} < 1$, as long as we keep the weight on the price, z, the same under the two market structures.

Second, however, the single price in the centralized market aggregates more information than each of the individual prices separately in the OTC market. Indeed, it is easy to check that $z_{CN} < z_V$ for any parameter values. This tends to make illiquidity higher in the OTC market. Note that increasing the ratio $\frac{z_V}{z_{CN}}$ increases illiquidity in the centralized market relative to the OTC market as

$$\frac{\partial \frac{\frac{1}{t_V}}{\frac{1}{t_{CN}}} |_{\frac{z_V}{z_{CN}}=x}}{\partial x} = \frac{\partial \frac{-\frac{2B}{n-1}\frac{1}{z}}{\frac{-nB}{n(xz-1)+2-zx}}}{\partial x} > 0.$$

As $\frac{z_V}{z_{CN}}$ is monotonically increasing in ρ, this force is strongest at the common value limit. As we prove in the proposition, this effect is sufficient to make illiquidity higher for in OTC market in the common value limit.

To understand the result on welfare, we start by comparing customers utility. Note first that the ratio of costumers' utility in the complete network OTC market and the centralized market is the ratio of the price variance in each market: $\frac{\frac{B}{n-1}\frac{n(n-1)}{2}E\left(p_{CN}^2\right)}{\frac{nB}{2}E\left(p_V^2\right)} = \frac{E\left(p_{CN}^2\right)}{E\left(p_V^2\right)}$. Also, in the common value limit, the price variance is larger under the OTC structure as

$$\lim_{\rho \to 1} \frac{E\left(p_{CN}^2\right)}{E\left(p_V^2\right)} = \lim_{\rho \to 1} \frac{\left(\frac{1}{2+z_{CN}}\right)^2 4\left(\mathcal{V}\left(e^i\right) + \mathcal{V}\left(e^i, e^j\right)\right)}{\left(\frac{1}{2+(n-1)z_V}\right)^2 2n\left(\mathcal{V}\left(e^i\right) + (n-1)\mathcal{V}\left(e^i, e^j\right)\right)} = \left(\frac{2n-3}{n-1}\right)^2 > 1.$$

As is apparent from the second expression above, there are two forces. On the one hand, in a centralized market the variance of the price is connected to the variance of the sum of all expectations, while on an OTC market it is connected to the variance of the sum of the two expectations at each link. The first one is higher, which makes costumers' expected utility higher on centralized markets. On the other hand, as $z_V > z_{CN}$, the multiplier coming from trading intensities tends to push customers' utility higher on the OTC market. The ratio $\frac{z_V}{z_{CN}}$ is maximal in the common value limit, and the second force turns out to dominate the first. So in this limit, the utility is higher under the OTC structure. As Panels A-D in Figure 2 demonstrate, when ρ is smaller, the first force might dominate, implying that utility tends to be larger under the centralized structure.

Finally, we explain why welfare is higher, but expected profit of dealers is lower in the OTC market in the common value limit. For this, let us rewrite the general formula for welfare, (29), as the sum of the value of allocations to dealers and customers

$$\sum_{ij}\left(E\left(q_{ij}^{i}\theta^{i}\right)+E\left(q_{ij}^{j}\theta^{j}\right)-\frac{\beta_{ij}}{2}E\left(p_{ij}\right)^{2}.\right) \tag{31}$$

Substituting in the closed-form expressions and taking the limit, it is easy to show that the sum of terms corresponding to dealers is actually larger in the OTC market than in the centralized market in the common value limit $\rho \to 1$. This is due to the larger trading intensity in OTC markets in this limit. The difference between formulas (29) and (31) represents, essentially, a transfer from dealers to customers. Since this transfer is larger under the OTC market structure, this explains the why welfare and profit move in opposite directions. As is apparent from the middle expression in (28), the total transfer is $\sum_{ij}\left(-\beta_{ij}E\left(p_{ij}^{2}\right)\right)$, twice the utility of customers, which, as we argued above, is indeed higher in the OTC complete network than in the centralized market in the common value limit.

5.3 The effect of more links: circulant networks with varying density

Panels A-D in Figure 2 also plot how welfare, customers' utility, dealers' profit and illiquidity compares in various (n,k)-circulants. With fewer links, welfare and customers' utility tends to decrease and illiquidity tends to increase, while dealers' profit might go either way.

As there are no explicit solutions for the conditional guessing game for circulant networks, we do not have analytical results for the circulant OTC networks either. Still, because of the symmetry, the intuition behind the numerical results is relatively simple. Decreasing the number of links in symmetric fashion has two main effects: each dealer learns less and each dealer has fewer opportunities to trade and intermediate. Learning less implies more concern about adverse selection, lower trading intensities on average, higher illiquidity and smaller variance of prices at each link (as fewer links implies lower variation in expectations as weights on the common prior increase and weights on signals decrease). Fewer opportunities to trade and smaller trading intensities imply smaller trading volume which is the dominating force in reduced welfare. The lower price variance implies reduced customers' utility and, by the logic explained above, smaller total transfer from dealers to customers. Profits can go either way, because the net effect of less trade and smaller transfers is ambiguous. As we see in the figure, close to the common value limit, less dense networks might be more profitable for dealers.

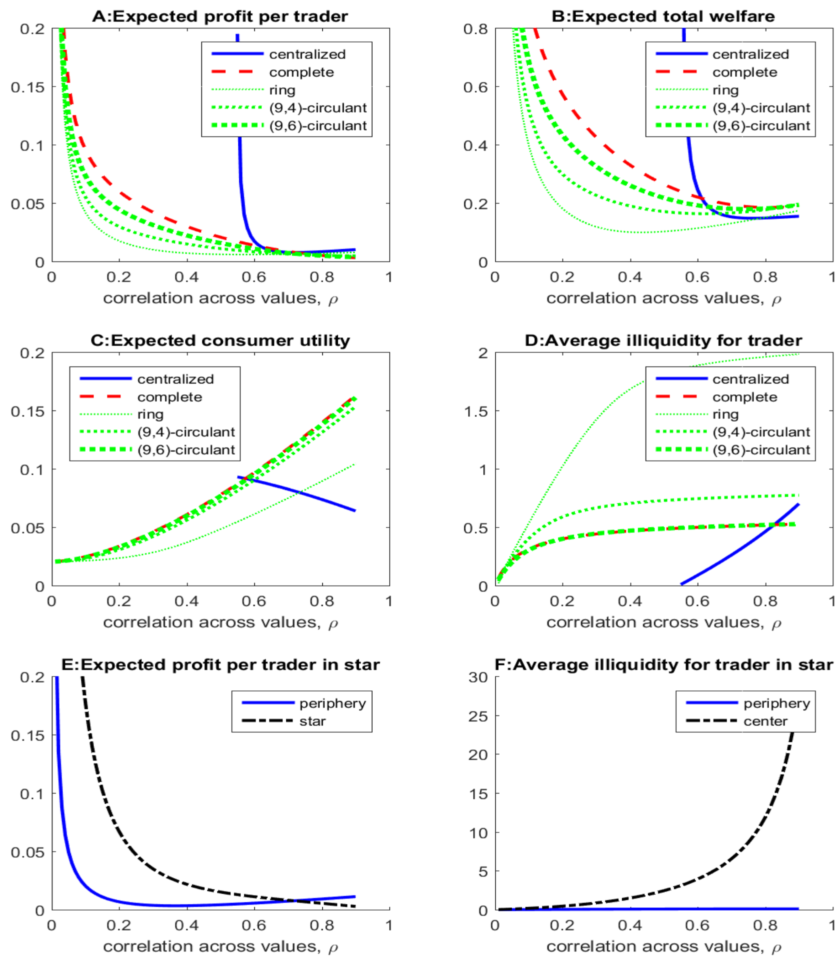

Figure 2: Expected profit, expected welfare, expected consumer utility, average trading cost (illiquidity) per trader in various networks. Parameters: $n = 9, B = 1, \sigma_\theta^2 = 0.1, \sigma_\varepsilon^2 = 1$.

5.4 The effect of asymmetry: periphery and the central dealer in a star network

The star network is an ideal case to study the effect of asymmetry on allocations and welfare. The main result in this part is that central agents do not always earn higher expected profit than periphery agents. In fact, expected profit is higher for periphery agents in the common value limit.

Simple, closed-form solutions that characterize the equilibrium in a star network are spelled out in Appendix B. The next proposition and Panels E-F in Figure 2 show analytical and numerical results concerning illiquidity, profit, and welfare.

Proposition 9 *In a star network the following statements hold*

1. *The adverse selection concern and the trading intensity of periphery traders are higher, $z_P > z_C$, $t_P > t_C$, or, equivalently, the central dealer faces a more illiquid market than the periphery dealers for any ρ.*

2. *In the common value limit, $\rho \to 1$, central dealer's profit converge to zero while the periphery dealer"s profit is bounded away from zero as $t_P \to -\beta$, $t_C \to 0$.*

We start by comparing the trading intensities, t_P and t_C. As we noted before, for the central agent prices are privately fully revealing. That is, her posterior belief is the same as the belief of dealers in a complete network or in a centralized market. In contrast, the learning of periphery dealers is limited by the fact that they observe a single price only. As a result, the weight that a periphery dealer puts on the price is larger than the weight the central dealer puts on the same price, so $z_P > z_C$ always holds. Intuitively, the periphery dealer is more concerned about adverse selection than the central dealer as the central dealer knows more. Therefore, from (22) the trading intensity of periphery traders is always larger as

$$\frac{t_P}{t_C} = \frac{2 - z_C}{2 - z_P} > 1.$$

Hence, at each link the central dealer trades with a smaller intensity, or, equivalently the market is less liquid for the central dealer than for the periphery dealer.

Importantly, the first part of Proposition 9 suggests a negative relationship between a dealer's cost of trading and the number of her counterparties' links. The star is a special case, in the sense that there is a strong negative relationship between the number of a dealer's links and the average number of her counterparties' links. Indeed, the assortativity coefficient in a star is minimal at -1. This is why, in the case of the star, this negative relationship translates into a higher cost of trading for central dealer. In contrast, in more general core-periphery networks the average number of links of more connected dealers' counterparties is often higher.[7] Indeed, in our calibrated simulations in Section 6 the assortativity coefficient

[7] Consider the example of the network of 15 dealers, where 5 central dealers are connected in a complete network, each of them is connected with one mid-level dealer, and each mid-level dealer is also connected with one periphery dealer.

tends to be positive, and, consistently, there is a negative relationship between number of own links and cost of trading.

Now we turn to profits and allocations in the star network. While the central dealer trades with less intensity, she also trades and intermediates across more links and, by (23), the distance between her expectation and the price is larger than for the periphery dealer. As is illustrated in Panels E-F in Figure 2, for a large set of parameter values, the effect of the smaller trading intensity is dominated, and trading as a central agent is more profitable in expectation than as a periphery agent. However, this is not always the case. As is apparent in the figure, this statement is reversed as we approach the common value limit. In fact, in the limit the expected profit of the central dealer is zero, while it is strictly positive for periphery dealers as we state in Proposition 9. Again, this is related to the strong negative assortativity in a star.

To see the intuition, it is illustrative to spell out how profits are determined close to the common value limit. In the limit, all dealers put diminishing weight on their own signal as they form expectations. Instead, in the conditional guessing game as $\rho \to 1$, periphery dealers put a weight of $\bar{z}_P \to 1$ on the expectation of the central dealer, while the central dealer puts equal weight on each of the periphery agents' expectations, implying $\bar{z}_C \to \frac{1}{n-1}$. Thus, by Proposition 9, as we approach the common value limit in the OTC game, this implies trading intensities of $t_P \to -\beta$, $t_C \to 0$. That is, central dealers do not trade in this limit at all, and periphery dealers trade only with customers. In the common value limit, the central agent has better information about the common value of the asset than periphery agents. So, as a manifestation of the no-trade theorem, there cannot be an equilibrium where these agents trade with each other. Therefore, the only remaining question is who trades with the customers. As periphery agents are more concerned about adverse selection, the price impact of the central dealer is larger. This implies that there is a price-quantity pair at which the central dealer stops trading, but at which the periphery dealer is still willing to trade. This results in positive trade between periphery and customers only.

6 Simulated OTC markets: Informational trades in realistic networks

An attractive feature of our model is that it generates a rich set of empirical predictions. As we emphasize in this section, for any given information structure and dealer network, our model generates the full list of demand curves and the joint distribution of bilateral prices and quantities, and measures of price dispersion, intermediation, trading volume, etc. Therefore, in principle, our results could be compared to established and to-be-established stylized facts from the growing empirical literature using transaction-level OTC data. This way, our model can be useful to determine whether dealers' asymmetric information explains stylized facts in particular markets, during certain episodes.

To illustrate this feature, we present some of the robust implications of our model for the relationship between the standard financial indicators, such as trading cost, price dispersion, size of trades, profitability, intermediation and characteristics of the dealer network. Our basic approach is to (1) spell out how these financial indicators and network characteristics should be related based on our theory; (2) show that these relationships hold in a sample of random networks calibrated to European CDS markets; (3) show that they remain robust as we change the characteristics of the underlying network.

We introduce the ingredients of this exercise in the following subsections.

6.1 Centrality, financial indicators, and informational efficiency

We start by defining the model equivalents of the financial indicators of interest building on the analysis in 5.1. We distinguish between dealer-level financial indicators and market-wide financial indicators.

The dealer-level financial indicators we consider are trading cost (or illiquidity), gross volume, intermediation, expected profit. As we explained above, a relevant measure of the cost of trading for i with dealer j is the slope of the inverse demand function of dealer j, $\frac{1}{t^i_{ij}}$. This is the price impact of dealer i in a transaction with dealer j, and this measure is closely and positively related to i's cost of a round-trip trade, the illiquidity i faces, and the mark-up or i's effective spread, measures that are often used in the empirical literature. To obtain a dealer-specific measure of the average trading cost or illiquidity for i, we average this slope across the trading partners of i, $\frac{1}{|g^i|}\Sigma_{ij\in g^i}\frac{1}{t^i_{ij}}$.

To measure gross volume for a given dealer, we use $E\left(\Sigma_{ij\in g^j}\left|q^i_{ij}\right|\right)$. For intermediation, we consider the absolute ratio of the expected gross trading volume to the expected net trading volume $\left|\frac{E\left(\Sigma_{ij\in g^i}|q^i_{ij}|\right)}{E\left(\Sigma_{ij\in g^i}q^i_{ij}\right)}\right|$ for a given dealer. Clearly, this ratio is always 1 for a dealer who has a single link to trade, but can be very large for dealers who trade a lot with their multiple trading partners, but many of their trades cancel each other.

As a measure of profitability, we use the expected profit of dealer i: $E\left(\Sigma_{ij\in g^i}q^i_{ij}\left(\theta^i - p_{ij}\right)\right)$.

The market-wide financial indicators we consider are two widely observed characteristics of OTC markets: price dispersion and the concentration of trade. For price dispersion, consider the correlation matrix of prices in each transaction, R_p. For simplicity, we use the minimum element of R_p as an inverse measure of price dispersion in a given network. For the concentration of trade, we use the Gini coefficient for gross volume.

In addition, we characterize informational inefficiency in a given market in two ways. First, at a dealer level, we measure the precision of a dealer's posterior (i.e., the inverse of the conditional variance) normalized by the precision posterior obtained by knowing the joint information set of all the dealers. At the market level, based on Section 4.2, we measure the percentage change in the expected sum of squared deviations of the private value and the posterior of a given dealer, $E\left[\sum_i\left(\theta^i - \mathcal{E}^i\left(s^i;\mathbf{e}_{g^i}\right)\right)^2\right]$, in our equilibrium compared with the planner's solution.

Given the insights of the analysis in Section 4 and 5, we expect that more connected dealer should learn more, intermediate more, trade larger gross volume at lower cost, and make more profit. (In the networks we simulate, we find a positive correlation across the number of links of a dealer and the number of links of her counterparty. In this case, the intuition we developed in Section 5.4 informs us that more central dealers should trade at a lower price impact.)

6.2 The baseline network

In all simulations, we work with the hybrid network formation model of Jackson and Rogers (2007). In particular, the network formation starts with a small complete network. Then, nodes are born sequentially. When a new node is born, m_R parents are randomly chosen from the existing nodes with uniform probability. The new node forms a link with any given parent with probability p. Then, from the set of existing connections of the parents, m_S nodes are chosen randomly. The new node forms links with any of such-selected neighbors of her parents with probability p. Hence, in expectation, each node forms $m \equiv p\,(m_S + m_R)$ connections where the ratio of uniformly selected to network based links is $r \equiv \frac{m_R}{m_S}$. We chose this framework as it can generate intermediate cases between pure core-periphery networks and networks with uniformly distributed links depending on the ratio of uniform to network-based links. It is parametrized by parameters m, p, r. When r is small, the resulting networks show core-periphery features, while when r is very large, the resulting network is close to a random network where each link is formed with equal probability. We follow the procedure described by Jackson and Rogers (2007) (see details in Appendix E), and calibrate these parameters to the European CDS market as described by Brunnermeier, Clerc and Scheicher (2013). In particular, to keep the exercise computationally manageable, we use their 54-dealer representation, focusing on the largest positions of the largest traders in the most important assets. The procedure gives us the baseline parameters of $r = \frac{1}{3}$, $m = 8$, $p = 0.3$.

6.3 Simulated standard errors and sensitivity analysis

Given the baseline parameters for the network formation process, we generate 50 random networks, each with 54 dealers. For each network we calculate three market-wide measures: price dispersion, concentration of trade, and informational efficiency. We use the informational parameters $\rho = 0.5$, $\sigma_\theta^2 = \sigma_\varepsilon^2 = 1$. At the same time, for each network we run five single variable regressions with dealers' expected profit, expected intermediation, average cost of trade, expected gross volume, and posterior precision as the dependent variables and dealers' centrality as the independent variable. Thus, we obtain 50 estimates for each market-wide measure and for each regression coefficient. We interpret the mean over the 50 estimates as a point-estimate for each market-wide measure and for each regression coefficient, given the network parametrization. We interpret the range of the coefficients as simulated confidence intervals around that point estimate. For example, if we drop the maximal and the minimal elements from each of the 50 estimates for a given measure, we would get confidence intervals of 96% percent. For transparency, we do not drop any coefficient estimate, but plot all.

As a sensitivity analysis for the shape of the network, we generate random networks with different r values and different p values, keeping the size of the network and the average number of links fixed. For each set of parameters, we regenerate the 50 random networks, recalculate the market-wide measures, and re-run the same regressions. Following the same steps as above, we obtain point-estimates for market-wide measure and each regression coefficient, together with simulated standard errors, for each parametrization of the network. We report only the results we obtain by varying r. The results obtained by varying p are reported in the Appendix E.

Observing the overlap of the ranges of the estimates as we vary the parameters is informative whether or not the variation in parameters has a significant effect on the estimates. For example, in panel B in Figure 3, the estimated concentration looks significantly larger when $r = \frac{1}{3}$ than when $r = 3$, as there is minimal overlap in the estimated measures.

Note also, that in the Appendix E, we also report how the coefficients in our regressions and our market-wide measures change as a function of other characterizations of the underlying network, e.g., clustering, assortativity and diameter.

6.4 Results

We present our results in panels A-H in Figure 3. The main lesson is that the simulated confidence intervals never include 0 for any of our slope coefficients in each of the regressions and treatments. That is, our observations that more connected dealers trade more, intermediate more, learn more, trade at lower cost and make more profits are robust to all our sensitivity checks. We also observe concentrated trades, dispersed prices and informational inefficiency across all treatments.

In what follows, we assess how our treatments affect the strength of these relationships.

Panels A and B in Figure 3 show the estimates of the market-wide measures of price dispersion and concentration of trading volume. Price dispersion appears to vary relatively little with the type of the network. In fact, the range of the estimated price dispersion measures largely overlap as r, the ratio of uniform to network-based links, changes, suggesting that this parameter does not have a strong effect on price dispersion. Concentration of gross volume is relatively high in all treatments. As expected, concentration is larger in a network with stronger core-periphery features.

Panels C-F in Figure 3 show the slope estimates in regressions with dealer degree centrality on the right hand side and expected profit, intermediation, trading cost, and gross volume on the left. Clearly, a dealer with more links tends to trade more, at lower cost, make more profit,

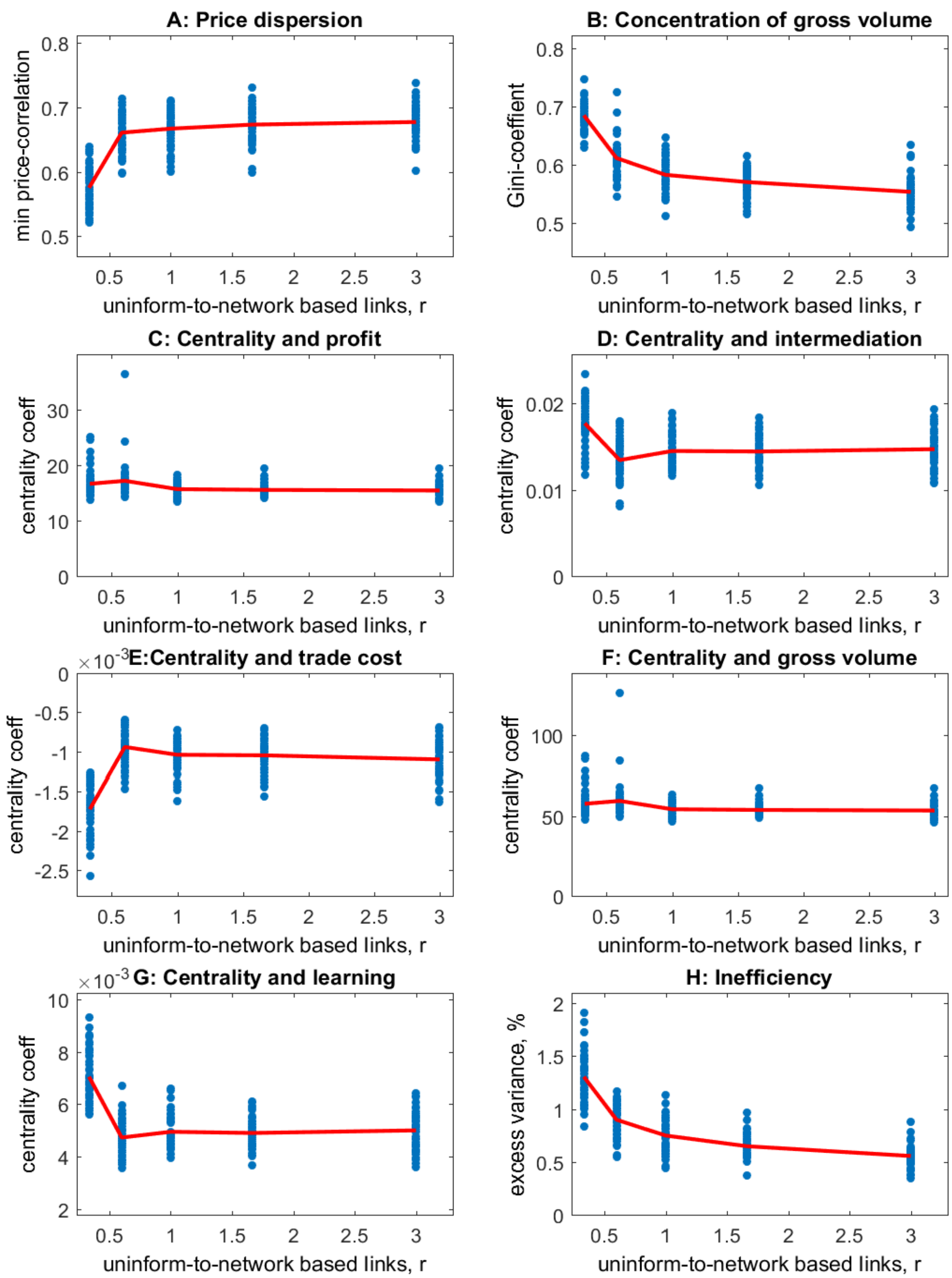

Figure 3: Panels A,B and H show the minimal element in the price correlation matrix, the Gini-coefficient of volume and excess variance compared to the planner's solution, while C-G show slope coefficients in regressions of expected profit, expected intermediation, average price impact, gross volume and learning on centrality on simulated networks. Each random network is generated by the method in Jackson and Rogers (2007) with varying uninform-to-network based links ratio. For a given r, 50 networks are generated. Each blue circle corresponds to a network realization, while the red line shows the average for a set of networks with fixed parameters. Parameters are $n = 54$, $m = 8$, $p = 0.3$,. $\sigma_\varepsilon^2 = \sigma_\theta^2 = 1$, $\rho = 0.5$.

and intermediate more. Also, this relationship does not seem to be affected by whether the network exhibits more or less core-periphery features.

Panels G-H in Figure 3 show that both the fact that the model implies informational inefficiency and the effect that dealers with more links learn more are robust to our treatments. Panel G suggests that the effect of r, the ratio of uniform to network-based links is unclear to the connection between centrality and learning except perhaps for very small r (for which the effect seems stronger). The inefficiency is larger when the network is closer to a core-periphery structure. This is because in a core-periphery network, there are agents with many links who learn a lot compared to the others. Thus, the fact that they do not internalize the potential social benefit of others' learning from their actions is more detrimental than in a network where such agents are less prevalent.

6.5 Simulation results and empirical findings

Because of data limitation, studies that connect dealer's network characteristics with economic indicators are rare. As an example, Li and Schürhoff (2014), consistent with our predictions, also show that central agents in the municipal bond market trade more, their trades are more profitable, and they seem to be better informed than others. The positive relationship between centrality and trading volume is also confirmed by Roukny, Georg and Battiston (2014) for a data-set of European CDSs. Hollifield, Neklyudov and Spatt (2012) shows that central agents in a securitized loan market intermediate more. Interestingly, these two studies find opposing patterns in terms of the relationship between trading cost (measured as mark-up) and network position. While Li and Schürhoff (2014) finds that in the municipal bond market more central dealers trade at higher mark-up, Hollifield, Neklyudov and Spatt (2012) finds that in the collateralized loan market more central dealers trade at lower mark-up. Our analysis suggests that differences in the assortativity of the underlying dealer network might be a simple explanation for these contrasting facts.

Note that thinking about the underlying trading network structure might be useful even when the econometrician has only limited information on dealers' characteristics. Indeed, given our results, we should expect that larger transaction size is associated to smaller cost of trading, more profitability per transactions, less dispersed prices across simultaneous transactions, but more volatile prices across time periods. The reason is that each of these characterize transactions of more connected dealers. From this group of predictions, the pattern that percentage cost is decreasing in the size of the transaction is a robust observation in many different contexts (see Green, Hollifield and Schurhoff (2007) and Li and Schürhoff (2014) on municipal bonds and Edwards, Harris and Piwowar (2007) and Randall (2015) on US corporate bonds).

Finally, consistently with our observations Atkeson, Eisfeldt and Weill (2012) and Hollifield, Neklyudov and Spatt (2012) report the CDS and the securitized loan markets are highly concentrated. While the same is true when US corporate bonds are evaluated in the aggregate, Schultz (2001) reports that trading in specific bonds seems to be spread across multiple dealers. Our model is silent on these differences.

7 An extension: Illiquidity and learning in segmented markets

Our model can be extended to provide insights about trade in segmented markets as well. Markets are segmented when investors, such as hedge funds and asset management firms, trade in some markets but not in others. Although segmented, markets can be connected, in the sense that agents are able to trade in multiple venues at the same time.

Formally, we consider an economy in which there are N trading posts connected in a network g. At each trading post, I, there exist n^I risk-neutral dealers. Each dealer $i \in I$ can trade with other dealers in her own trading post and with dealers at any trading post J that is connected with the trading post I by a link IJ. Essentially, the link IJ represents a trading venue in which dealers at trading posts I and J can trade with each other. However, they have access to trade in other venues at the same time.

Apart from the market structure, the set-up is unchanged. As before, each dealer's trading strategy is a generalized demand function that maps the prices that prevail in markets she participates in, into a vector of quantities she wishes to trade in each market. We still assume that each dealer has a mass of B customers implying that the absolute slope of customers' demand in each trading venue is $-\beta_{IJ} = \frac{B(Nn_P+n_C)}{N}$.

The general set-up is described in detail in Appendix C. While this extension enriches the analysis in an important dimension, it comes with a loss of tractability. The main technical difficulty that arises when markets are segmented is that the price in each trading venue IJ is a linear combinations of all dealers beliefs located at trading post I and J, as shown in equation

(C.11). This implies that the signals of dealers in the same trading post obscure the (sum of) beliefs of the dealers in neighboring trading posts, such that a dealer can no longer invert the prices she observes and infer her neighbors posteriors.

Here, we focus the discussion on illustrating the effects of market integration on learning from prices and market liquidity. For this, we restrict ourselves to considering a star network, in which there are n_P dealers at each periphery trading post, and n_C dealers at the central trading post. In particular, we conduct the following numerical exercise. We consider an economy with nine agents. Keeping their information set fixed, we compare the following four market structures:

1. 8 trading posts connected in a star network, with one agent in each trading post ($N = 8$, $n_P = 1$, $n_C = 1$), that is, 8 trading venues. This is our baseline model with a star network.

2. 4 trading posts connected in a star network, with two agents in each periphery node and one agent in the central node ($N = 4$, $n_P = 2$, $n_C = 1$), that is, 4 trading venues.

3. 2 trading posts connected in a star network, with four agents in each periphery node and one agent in the central node ($N = 2$, $n_P = 4$, $n_C = 1$), that is, 2 trading venues.

4. A centralized market ($N = 1$, $n_P = 9$, $n_C = 0$), that is, a single trading venue.

We consider two directions. First, we investigate what drives the illiquidity central and periphery agents face for changing degrees of market segmentation. We concentrate on (il)liquidity as this is a more commonly reported variable in the empirical literature, and we leave the analysis of welfare and expected profits to Appendix C. Second, to complement the analysis in Section 4 , we also analyze how much dealers can learn from prices under these market structures.

The left and center panels in Figure 4 show the average illiquidity that a periphery, $\frac{1}{t_P}$, and a central dealer, $\frac{1}{t_C}$, face in each of the scenarios described above. We also plot the average illiquidity that any agent in a centralized market, $\frac{1}{t_V}$, faces. For easy comparison, all the parameters are the same as in Section 5.

To highlight the intuition, we start with the extreme cases of market segmentation comparing illiquidity under a star network and in a centralized market.

7.1 Extreme cases of market segmentation

In this part, we compare illiquidity of dealers in a centralized market and that of a periphery or central dealer in a star network.

The solid curve in Panels D and the curves in panel F in Figure 2 illustrate that compared to any agent in a centralized market, the central agent in the star faces higher trading price impact in general, but the periphery agents tend to face smaller price impact when the correlation across values is sufficiently high. We partially prove this result**.** The following proposition states that if ρ is sufficiently large, illiquidity for the central agent is larger, while illiquidity for the periphery agents is lower than that for an agent in a centralized market and, when ρ is sufficiently small, illiquidity for any agent in a star network is larger than the illiquidity for any agent in a centralized market.

Proposition 10

1. *When ρ is sufficiently small, such that z_V is sufficiently close to $1-\frac{1}{n-1}$, then illiquidity for any agent in a star network is larger than for any agent in a centralized market*

2. *In the common value limit, when $\rho \to 1$,*

   (a) *illiquidity for a central agent is higher in a star network than for any agent in a centralized market, and*

   (b) *illiquidity for a periphery agent is lower in a star network than for any agent in a centralized market.*

Similarly to the comparison between the complete OTC network and the centralized market in Section 5.2, there are two main forces that drive the illiquidity ratios $\frac{t_V}{t_C}$ and $\frac{t_V}{t_P}$. First, the best response function **(30)** of a dealer in a centralized market is steeper and has a larger intercept than the best response function (26) of central and periphery dealers in the star OTC network. Simple algebra shows that if, counterfactually, the adverse selection parameters were equal, $z_P = z_C = z_V$ then $\frac{t_V}{t_C}|_{z_V=z_C=z_P} = \frac{t_V}{t_P}|_{z_V=z_C=z_P} > 1$, that is, illiquidity for any agents in the OTC market would be higher than for any agent in the centralized market. This is the effect which dominates when ρ is small.

Second, parameters z_C, z_V and z_P differ. As we stated in Proposition 9 central agents face less liquid markets than periphery agents, $\frac{1}{t_P} < \frac{1}{t_C}$ because periphery agents are more concerned about adverse selection ($z_C < z_P$). This implies that $\frac{t_V}{t_C} > \frac{t_V}{t_P}$ and difference is increasing for higher ρ. In fact, in the common value the central agent faces an infinitely illiquid market in the sense that $t_C \to 0$, but consumers provide a relatively liquid trading environment for periphery agents. For periphery agents this is sufficiently strong to reduce their price impact below the centralized market level as stated in the second part of the proposition.

7.2 Intermediate cases of market segmentation

Interestingly, while the illiquidity a central agent faces is monotonic in segmentation, the illiquidity a periphery agents face is not. We see in left panel of Figure 4 is how the relative strength of the two forces identified in Section 7.1 plays out in the four scenarios we consider. First, related to the effect of decentralization on best response functions, illiquidity for any agent decreases as the market structure approaches a centralized market. Second,the effect coming from the differing weights of z_C and z_P is weaker in more centralized markets. The reason is that as central dealers observe less prices in more centralized markets, they put a larger weight, z_C in each price, implying a smaller difference between z_P and z_C. This is the reason why the illiquidity a periphery agent faces under the 2 trading venues structure increases with ρ almost as fast as in centralized markets. With 4 venues the effect of ρ is weaker.

Turning to the effect of segmentation on learning, note that for the central dealer prices are fully revealing under any of the segmented market structures in this exercise. This is because each price she observes is a weighted sum of her own signal and the sum of signals of the periphery dealers trading in each venue. Hence, the prices the central dealer observes represent a sufficient statistic for all the useful information in the economy. This would not be the case if there were more than one dealer at the central trading post.

In contrast, as it is shown in the right panel of Figure 4 a periphery agent in a segmented market always learns less than the central agent, or any agent in a centralized market. Interestingly, for small correlation across values, ρ, a periphery agent in a more segmented market learns more, while for a sufficiently large correlation across values the opposite is true. The intuition relies on the relative strength of opposing forces. The price a periphery agent learns from is a weighted average of the sum of posteriors of periphery agents in the same trading

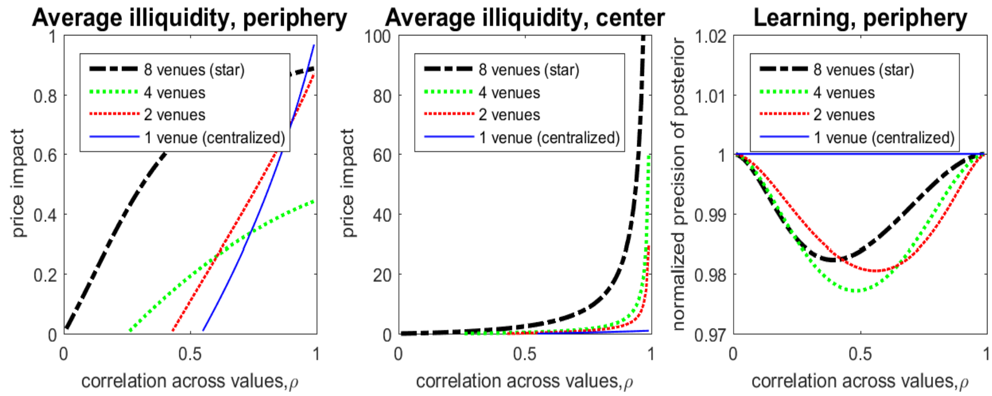

Figure 4: Illiquidity on segmented markets. We show our measure of illiquidity for central agents , $\frac{1}{t_C}$, (left panel) and for periphery agents, $\frac{1}{t_P}$, (right panel) when there are 8 trading venues (dotted), 4 trading venues (dashed), 2 trading venues (dash-dotted), and in the centralized market (solid) as a function of the correlation across values, ρ. Other parameter values are $\sigma_\theta^2 = 1$, $\sigma_\varepsilon^2 = 0.1$, $B = 1$.

post and the posterior of the central agent. The posterior of the central agent is more informative than any of the posteriors that periphery agent at the same trading post have. The more segmented the market is, the easier is for a dealer at a periphery trading post to isolate the posterior of the central dealer (for example, in the baseline star network, any price reveals the posterior of the central dealer perfectly). At the same time, the sum of the posteriors of periphery dealers at a periphery trading post is more informative in a less segmented market, as the noise in the signal, as well as the private value components tend to cancel out. This latter effect helps learning more when the private value component is more important, that is, when ρ is small. This explains the pattern in the right panel of Figure 4.

8 Conclusions

In this paper, we have proposed to model trading and information diffusion in OTC markets. Dealers trade on a fixed network, and each dealer's strategy is represented as a quantity-price schedule. We showed that information diffusion through prices is unaffected by dealers' strategic trading motives, that each price partially incorporate the private information of all dealers, and we identified an informational externality constraining the informativeness of prices. We also highlighted that trade decentralization per se can both increase or decrease welfare and that the main determinant of a dealer's trading cost is not her centrality but the centrality of her counterparties. We use extensive simulations to illustrate that more central dealers tend to learn more, trade more at lower costs and earn higher expected profit.

Importantly, trading protocols in OTC markets have become increasingly diverse. Certain developments in trading protocols improve the fit of our demand submission game. For example, in some OTC derivatives markets clients of dealers can request "dealer runs" providing a menu of potential trades (Duffie (2012*a*)). However, there are also a number of other protocols (e.g. broker assisted work-up protocols) which our paper does not cover. Given this increasing diversity, it is important to develop frameworks which put limited emphasis on any one particular trading protocol, and still can capture robust features of OTC markets. Our approach emphasizes that links are persistent, that market structure is concentrated, and that dealers intermediate trade between otherwise disconnected counterparties. Our model yields price-quantity pairs which are consistent with each dealers information, potential trading partners and objectives. We implicitly suggest that if such pairs exists, it is likely that the market will converge to these points, independently of the trading protocol.

Demand and supply curves have been a powerful tool to model equilibrium in centralized good markets since the beginning of economic thinking. With our approach of generalized demand curves on networks, we have found a way to generate insights for decentralized markets as well.

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A Appendix: Proofs

In some proofs we use the convention that a network, g, can be represented by an *adjacency* matrix $A(g)$ with elements $A_{ij} = 1$ if $ij \in g$, and $A_{ij} = 0$ if $ij \notin g$.

Proof of Proposition1

We prove the statement in a more general form than stated. It proves existence for a general Gaussian information structure. We need only that if $\boldsymbol{\omega}^i$ is a vector of the covariances between θ^i and each of the signals (in our case it is $\omega_{ii} = \sigma_\theta^2$ and $\omega_{ij} = \rho\sigma_\theta^2$ for $i \neq j$), then $\boldsymbol{\omega}^i > 0$ for all i. Note that we can rewrite the problem as follows. We are looking for a V^* matrix of which each column $\mathbf{v}^i$ is the solution of the problem of

$$\begin{aligned} &\max_{\mathbf{v}^i} \left(2\left(\mathbf{v}^i\right)^\top \boldsymbol{\omega}^i - \left(\mathbf{v}^i\right)^\top \Sigma \mathbf{v}^i\right) \\ &\text{s.t. } \mathbf{v}^i = \left[\bar{\mathbf{y}}^i + V^* \bar{\mathbf{z}}^i\right] \\ &z_{ij}^i = 0 \Longleftrightarrow A_{ij} = 0 \end{aligned} \tag{A.1}$$

where Σ is the covariance matrix of signals, $\mathbf{s}$, and $\bar{\mathbf{y}}^i$ is a column vector with $\bar{y}^i$ at the $i-th$ place and 0 otherwise.

To see that this V^* exists, let us first define the matrix mapping $F : R^{n\times n} \to R^{n\times n}$, which maps any $n \times n$ matrix V^0 to another one with columns $\mathbf{v}^i$ defined by

$$\begin{aligned} &\mathbf{v}^i \equiv \arg\max_{\mathbf{v}^i} \left(2\left(\mathbf{v}^i\right)^\top \boldsymbol{\omega}^i - \left(\mathbf{v}^i\right)^\top \Sigma \mathbf{v}^i\right) \\ &\text{s.t. } \mathbf{v}^i = \left[\bar{\mathbf{y}}^i + V^0 \bar{\mathbf{z}}^i\right] \\ &z_{ij}^i = 0 \Longleftrightarrow A_{ij} = 0 \end{aligned} \tag{A.2}$$

Further, let

$$\mathbb{V}^i \equiv \{\mathbf{v}^i : \left(\mathbf{v}^i\right)^\top \Sigma \mathbf{v}^i - 2\left(\mathbf{v}^i\right)^\top \boldsymbol{\omega}^i + \frac{\omega_{ii}^2}{\Sigma_{ii}} \leq 0\}.$$

and $\mathbb{V}^{n\times n} \equiv \mathbb{V}^1 \times \mathbb{V}^2 .. \times \mathbb{V}^n$ be the set of matrices with columns $\mathbf{v}^i \in \mathbb{V}^i$.

We need to show that F is a self map with respect to the set of matrices $\mathbb{V}^{n\times n}$ and that $\mathbb{V}^{n\times n}$ is a convex compact set. Hence, the Brouwer fixed-point theorem applies. We proceed in steps.

1. We show that F defined by (A.2) is a self-map.

   For this, note that increasing the number of 0-s in the $i-th$ row of A (decreasing the number of links to i in the network) adds more constraints to the problem (A.2). So we consider the extreme problem where the $i-th$ row and column of A has only zeros, that is, each $z_{ij}^i \equiv 0$. It is easy to show that in this case the problem reduces to

   $$\frac{\omega_{ii}^2}{\Sigma_{ii}} = \max_{v_{ii}} \left[2v_{ii}\omega_{ii} - v_{ii}\Sigma_{ii}v_{ii}\right]$$

   with a solution of $v_{ii} = y^i = \frac{\omega_{ii}}{\Sigma_{ii}}$ and $v_{ij} = 0$ for all $i \neq j$. Thus, for any A with non-zero elements in the $i-th$ row and column, $\frac{\omega_{ii}^2}{\Sigma_{ii}}$ is a lower bound on the value agent i can achieve, that is, the solution $\mathbf{v}^i$ will satisfy $\frac{\omega_{ii}^2}{\Sigma_{ii}} \leq 2\left(\mathbf{v}^i\right)^\top \boldsymbol{\omega}^i - \left(\mathbf{v}^i\right)^\top \Sigma \mathbf{v}^i$. Implying that for any V_0 and A, F projects to $\mathbb{V}^{nxn}$.

2. Given that the Cartesian product of convex and compact sets is also convex and compact, we only have to show that each $\mathbb{V}^i$ is convex, closed and bounded

   (a) $\mathbb{V}^i$ is convex. Under the assumption that Σ is positive definite, $\left(\mathbf{v}^i\right)^\top \Sigma \mathbf{v}^i - 2\left(\mathbf{v}^i\right)^\top \boldsymbol{\omega}^i$ is a convex function (the sum of a convex and a linear function). From the fact that the sub-level sets of a convex functions are convex, it follows that the set $\mathbb{V}^i$ is convex.

   (b) $\mathbb{V}^i$ is closed. Let $g(\mathbf{v}^i) = \left(\mathbf{v}^i\right)^\top \Sigma \mathbf{v}^i - 2\left(\mathbf{v}^i\right)^\top \boldsymbol{\omega}^i + \frac{\omega_{ii}^2}{\Sigma_{ii}}$ be a function from R^n to R. Clearly, g is continuous and $\mathbb{V}^i \equiv \{\mathbf{v}^i : g(\mathbf{v}^i) \leq 0\}$. Let $\mathbf{v}_n^i$, $n = 1...\infty$ be a convergent series of vectors in $\mathbb{V}^i$ with $\mathbf{v}_\infty^i$ being the limit point of this series. Since g is continuous, we have $g(\mathbf{v}_\infty^i) = \lim_{n\to\infty} g(\mathbf{v}_\infty^i) \leq 0$. Hence $\mathbf{v}_\infty^i \in \mathbb{V}^i$.

   (c) $\mathbb{V}^i$ is bounded. Note that the function $g(\mathbf{v}^i)$ is strictly convex, continuous, and twice-differentiable. Hence, there exists a minimum $\mathbf{v}_{\text{min}}^i$ that $g(\mathbf{v}_{\text{min}}^i) \leq g(\mathbf{v}^i)$ for all $\mathbf{v}^i \in \mathbb{V}^i$ Also, from the definition $\mathbb{V}^i$, $g(\mathbf{v}^i) \leq 0$ for all $\mathbf{v}^i \in \mathbb{V}^i$. Note also that $g(\cdot)$ is strongly convex on $\mathbb{V}^i$ as there exists $m > 0$ such that $\nabla^2 g(\mathbf{v}) - m\mathbf{I} = 2\Sigma - m\mathbf{I}$ is positive definite (for example, one can pick $m = \sigma_\theta^2 + \sigma_\varepsilon^2$). Also, from strong convexity

   $$g(\mathbf{v}'') \geq g(\mathbf{v}') + \nabla g(\mathbf{v}')^\top(\mathbf{v}'' - \mathbf{v}') + \frac{m}{2}||\mathbf{v}'' - \mathbf{v}'||_2^2.$$

   for any $\mathbf{v}', \mathbf{v}'' \in R^n$. In particular, for $\mathbf{v}' = \mathbf{v}_{\text{min}}^i$, we have $\nabla g(\mathbf{v}_{\text{min}}^i) = 0$ implying that

   $$g(\mathbf{v}'') - g(\mathbf{v}_{\text{min}}^i) \geq \frac{m}{2}||\mathbf{v}'' - \mathbf{v}_{\text{min}}^i||_2^2.$$

   Let us pick $\mathbf{v}'' = \mathbf{v}^i$ an arbitrary element of $\mathbb{V}^i$. Then $g(\mathbf{v}'') \leq 0$ implying

   $$-\frac{2}{m} g(\mathbf{v}_{\text{min}}^i) \geq ||\mathbf{v}^i - \mathbf{v}_{\text{min}}^i||_2^2$$

   proving the claim.

Proof of Proposition 2 and Corollary 1

Consider an equilibrium of the conditional guessing game in which

$$E(\theta^i \left| s^i, \mathbf{e}_{g^i} \right.) = \bar{y}^i s^i + \sum_{k \in g^i} \bar{z}_{ik}^i E(\theta^k \left| s^k, \mathbf{e}_{g^k} \right.)$$

for every i. If the system (21) has a solution, then

$$E(\theta^i \left| s^i, \mathbf{e}_{g^i} \right.) = \frac{y^i}{\left(1 - \sum_{l \in g^i} z_{il}^i \frac{2 - z_{li}^l}{4 - z_{il}^i z_{li}^l}\right)} s^i + \sum_{k \in g^i} z_{ik}^i \frac{\frac{2 - z_{ik}^i}{4 - z_{ik}^i z_{ki}^k}}{\left(1 - \sum_{l \in g^i} z_{il}^i \frac{2 - z_{li}^l}{4 - z_{il}^i z_{li}^l}\right)} E(\theta^k \left| s^k, \mathbf{e}_{g^k} \right.) \quad \text{(A.3)}$$

holds for every realization of the signals, and for each i. Now we show that choosing the prices and demand functions (23) and (24) is an equilibrium of the OTC game.

First note that (26) for i and j at a given link implies (22). Also, the choice (24) implies

$$E(\theta^i \left| s^i, \mathbf{e}_{g^i} \right.) = y^i s^i + \sum_{k \in g^i} z^i_{ik} p_{ij} = E(\theta^i \left| s^i, \mathbf{p}_{g^i} \right.). \tag{A.4}$$

The second equality comes from the fact that the first equality holds for any realization of signals and the projection theorem determines a unique linear combination with this property for a given set of jointly normally distributed variables. Thus, (24) for each ij link is equivalent with the corresponding first order condition (10). Finally, (A.4) also implies that the bilateral clearing condition between a dealer i and dealer j that have a link in network g

$$t^i_{ij}\left(E(\theta^i \left| s^i, \mathbf{p}_{g^i} \right.) - p_{ij}\right) + t^j_{ij}\left(E(\theta^j \left| s^j, \mathbf{p}_{g^j} \right.) - p_{ij}\right) + \beta_{ij} p_{ij} = 0$$

is equivalent to (23). That concludes the statement.

Corollary 1 follows from the direct observation of (23) and (24) and (22).

Proof of Proposition 3

Case 1: Circulant networks

We provide here a draft of the proof. Further details of each step is available on request.

Step 1: We show that for circulant networks, each $\bar{z}^i_{ij} > 0$ for any $\rho \in (0, 1)$.

1. First, we show that $\bar{z}^i_{ij} > 0$ in the limit $\rho \to 0$. Clearly when $\rho = 0$, the equilibrium V is a diagonal matrix as the signals of others are uninformative in this pure private value case. The starting point is to show that for diminishingly small ρ, the off-diagonal elements of V which are corresponding to first neighbors are diminishing at a slower rate than the rest of the off-diagonal elements. In particular, we conjecture and verify that there are constants a_0 and a_1 that
$$\lim_{\rho \to 0}\left(\frac{V - a_0 I}{\rho}\right) = a_1 A$$
where I and A are the identity matrix and the adjacency matrix respectively. For this, we calculate the matrices $\bar{Y}$ and $\bar{Z}$ which correspond to a starting matrix $V^0 = a_0 I + a_1 A$ for a given a_0 and a_1 in problem (A.2), obtain the resulting new matrix $V^1 = \left(\bar{Y} + \bar{Z}V^0\right)$, observe that each non-zero element in $\bar{Z}$, $\bar{z}^i_{ij} > 0$ are positive, and verify there are indeed a_0 and a_1 values for which $\lim_{\rho \to 0}\left(\frac{V^1 - a_0 I}{\rho}\right) = a_1 A$.

2. Given that all $\bar{z}^i_{ij}$ are positive in this limit, let us counterfactualy assume that there is $\rho \in (0, 1)$ for which at least one $\bar{z}^i_{ij} < 0$. By continuity, then must be a ρ_0 for which all $\bar{z}^i_{ij} \geq 0$ but at least one of them is zero. But this implies that for these parameters dealer i finds the expectation of one of her neighbors uninformative. Let $\{i_k\}_{k=1,\dots,m^i}$ be the set of i's neighbors and, without loss of generality, suppose that the index of this neighbor is m^i. The only way this holds is that there is a linear combination of s^i and $\left\{e^{i_k}\right\}_{k=1,\dots,(m^i-1)}$

which replicates $e^{i_{m^i}}$, that is, that there is an arbitrary vector $[\lambda_0, \lambda_1..\lambda_{m^i-1}]$, that

$$\lambda_0 s^i + \lambda_1 e^{i_1} + ...\lambda_{m^i-1} e^{i_{(m^i-1)}} = e^{i_{m^i}} \tag{A.5}$$

(a) Note that if the network is circulant, there must be an equilibrium where V is also circulant. To see this, note that problem (A.2) maps circulant networks into circulant networks. Also, given that we prove the properties of $\mathbb{V}^{n\times n}$ vector-by-vector in the proof of 1, repeating those steps proves the existence of a circulant V fixed point. Furthermore, in this equilibrium the rows corresponding to the expectation of agent i and j has to have the structure of $v_{i(i+l)} = v_{i(i-l)} = v_{j(j+l)} = v_{j(j-l)}$ for every $l \geq 0$ as long as $n \geq i-l, i+l, j-l, j+l \geq 1$. This is, the weight of each signal in the equilibrium expectation of a given dealer can depend only on whether that signal belongs to a first neighbor, or a second neighbor etc. of the given dealer. This is coming from the symmetry across dealers in circulant networks and the symmetric informational content of their expectations in this equilibrium.

(b) However, given this symmetric structure of the equilibrium V matrix, there are no v_{ij} and $[\lambda_0, \lambda_1..\lambda_{m^i-1}]$ values which can solve the equations (A.5) unless all v_{ij} are the same. For instance, let us spell this out the implied equation system for the first agent in a $(7,4)$ circulant network with $\bar{k}$ being the second neighbor. If the row of V corresponding to the expectation of the first neighbor of 1 has the structure of $v_1 \;\; v_0 \;\; v_1 \;\; v_2 \;\; v_3 \;\; v_3 \;\; v_2$ then his second neighbor must have the structure of $v_2 \;\; v_1 \;\; v_0 \;\; v_1 \;\; v_2 \;\; v_3 \;\; v_3$. Thus, we need

$$\lambda_0 \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} + (\lambda_1 + \lambda_2) \begin{pmatrix} v_1 \\ v_0 \\ v_1 \\ v_2 \\ v_3 \\ v_3 \\ v_2 \end{pmatrix} = (1 - \lambda_3) \begin{pmatrix} v_2 \\ v_1 \\ v_0 \\ v_1 \\ v_2 \\ v_3 \\ v_3 \end{pmatrix}$$

to hold for some scalars. It is easy to check that this implies that all $v-s$ are identical. However, it is also easy to check that a V with identical elements cannot be a fixed point.

This is a contradiction which concludes step 1.

Step 2: We show that $\bar{z}^i_{ij} < 1$for any $\rho \in (0,1)$.

For this, note that by using forward induction on the fixed point equation $V = \bar{Y} + \bar{Z}V$, we obtain that the equilibrium matrix V must satisfy

$$V = \bar{Y} \lim_{u\to\infty} \sum_0^u \left(\bar{Z}\right)^u + \lim_{u\to\infty} \left(\bar{Z}\right)^{u+1} V.$$

As $\rho \in (0,1)$ the diagonal of $\bar{Y}$ must be strictly positive, as s^i must contain residual information on the private value element of θ^i relative to the guesses of others. We know from Proposition 1 that V exists. From the fact that all elements of $\bar{Z}$ are non-negative and from the fact that the Neumann series $\lim_{u\to\infty}\sum_0^u \left(\bar{Z}\right)^u$ converges if and only if $\lim_{u\to\infty}\left(\bar{Z}\right)^{u+1}=0$ (see Meyer (2000) page 618), we must have that indeed $\lim_{u\to\infty}\left(\bar{Z}\right)^{u+1}=0$. As $\bar{Z}$ must be symmetric for a circulant network, and all elements are non-negative, if any elements were larger than 1, then there were some elements of $\lim_{u\to\infty}\left(\bar{Z}\right)^{u+1}$ which would not diminish (as the elements $\left(\bar{z}_{ij}^i\right)^{u+1}$ will be a component in some elements of the the matrix $\left(\bar{Z}\right)^{u+1}$ for any i and j).

Step 3: Now, we search for equilibria such that beliefs are symmetric, that is

$$z_{ij}^i = z_{ji}^j$$

for any pair ij that has a link in network g.

The system (21) becomes

$$\begin{aligned}
\frac{y^i}{\left(1-\sum\limits_{k\in g^i} z_{ik}^i \frac{2-z_{ik}^i}{4-\left(z_{ik}^i\right)^2}\right)} &= \bar{y}^i \\
z_{ij}^i \frac{\frac{2-z_{ij}}{4-z_{ij}^2}}{\left(1-\sum\limits_{k\in g^i} z_{ik}^i \frac{2-z_{ik}^i}{4-\left(z_{ik}^i\right)^2}\right)} &= \bar{z}_{ij}^i
\end{aligned}$$

for any $i\in\{1,2,...,n\}$. Working out the equation for z_{ij}^i, we obtain

$$\frac{z_{ij}^i}{2+z_{ij}^i} = \bar{z}_{ij}^i\left(1-\sum_{k\in g^i}\frac{z_{ik}^i}{2+z_{ik}^i}\right)$$

and summing up for all $j\in g^i$

$$\sum_{j\in g^i}\frac{z_{ij}^i}{2+z_{ij}^i} = \sum_{j\in g^i}\bar{z}_{ij}^i\left(1-\sum_{k\in g^i}\frac{z_{ik}^i}{2+z_{ik}^i}\right).$$

Denote

$$S^i \equiv \sum_{k\in g^i}\frac{z_{ik}^i}{2+z_{ik}^i}.$$

Substituting above and summing again for $j\in g^i$

$$S^i\left(1+\sum_{j\in g^i}\bar{z}_{ij}^i\right) = \sum_{j\in g^i}\bar{z}_{ij}^i$$

or

$$S^i = \frac{\sum\limits_{j\in g^i}\bar{z}_{ij}^i}{\left(1+\sum\limits_{j\in g^i}\bar{z}_{ij}^i\right)}.$$

We can now obtain

$$z_{ij}^i = \frac{2\bar{z}_{ij}^i\left(1 - S^i\right)}{1 - \bar{z}_{ij}^i\left(1 - S^i\right)} \tag{A.6}$$

and

$$y^i = \bar{y}^i\left(1 - S^i\right).$$

Finally, the following logic show that $z_{ij}^i \leq 2$. As $\bar{z}_{ij}^i < 1$, $2\bar{z}_{ij}^i < \left(1 + \sum_{j \in g^i} \bar{z}_{ij}^i\right)$ implying that $2\bar{z}_{ij}^i\left(1 - S^i\right) < 1$ or $2\bar{z}_{ij}^i\left(1 - S^i\right) < 2\left(1 - \bar{z}_{ij}^i\left(1 - S^i\right)\right)$, which gives the result by A.6.

Case 2: Star networks

We give the closed-form solutions for the star network in Appendix B.2. One can check by straightforward algebra that the resulting z_{ij}^i are indeed in the $[0, 2]$ interval.

Proof of Proposition 4

In an equilibrium of the OTC game, prices and quantities satisfy the first order conditions (10) and must be such that all bilateral trades clear.

Since market clearing conditions (6) are linear in prices and signals, we know that each price (if an equilibrium price vector exists) must be a certain linear combination of signals. Thus, each price is normally distributed.

From the first order conditions we have that

$$q_{ij}^i(s^i, \mathbf{p}_{g^i}) = t_{ij}^i\left(E(\theta^i \left| s^i, \mathbf{p}_{g^i}\right.) - p_{ij}\right).$$

The bilateral clearing condition between a trader i and trader j that have a link in network g implies that

$$t_{ij}^i\left(E(\theta^i \left| s^i, \mathbf{p}_{g^i}\right.) - p_{ij}\right) + t_{ij}^i\left(E(\theta^j \left| s^j, \mathbf{p}_{g^j}\right.) - p_{ij}\right) + \beta_{ij}p_{ij} = 0$$

and solving for the price p_{ij} we have that

$$p_{ij} = \frac{t_{ij}^i E(\theta^i \left| s^i, \mathbf{p}_{g^i}\right.) + t_{ij}^i E(\theta^j \left| s^j, \mathbf{p}_{g^j}\right.)}{t_{ij}^i - \beta_{ij}}$$

Since agent i knows $E(\theta^i \left| s^i, \mathbf{p}_{g^i}\right.)$, by definition, the vector of prices $\mathbf{p}_{g^i}$ is informationally equivalent for her with the vector of posteriors of her neighbors $\mathbf{E}_{g^i} = \left\{E(\theta^j \left| s^j, \mathbf{p}_{g^j}\right.)\right\}_{j \in g^i}$. This implies that

$$E(\theta^i \left| s^i, \mathbf{p}_{g^i}\right.) = E(\theta^i \left| s^i, \mathbf{E}_{g^i}\right.).$$

Note also that as each price is a linear combination of signals and $E\left(\theta^j | \cdot\right)$ is a linear operator on jointly normal variables, there must be a vector $\mathbf{w}^i$ that $E(\theta^i \left| s^i, \mathbf{p}_{g^i}\right.) = E(\theta^i \left| s^i, \mathbf{E}_{g^i}\right.) = \mathbf{w}^i\mathbf{s}$. That is, the collection of $\left\{\mathbf{w}^i\right\}_{i=1,\ldots n}$ has to satisfy the system of n equations given by

$$\mathbf{w}^i\mathbf{s} = E(\theta^i \left| s^i, \left\{\mathbf{w}^j\mathbf{s}\right\}_{j \in g^i}\right.)$$

for every i. However, the collection $\left\{\mathbf{w}^i\right\}_{i=1,\ldots n}$ that is a solution of this system, is also an equilibrium of the conditional guessing game by construction.

Proof of Proposition 5

Equation (23), the fact that $t^i_{ij} > 0$ for all i and j, and $E(\theta^i \left| s^i, \mathbf{p}_{g^i} \right.) = E(\theta^i \left| s^i, \mathbf{E}_{g^i} \right.) = \mathbf{v}^i\mathbf{s}$ from Proposition 4 implies that we only have to show that all elements of the equilibrium V matrix defined in the proof of Proposition 1 are strictly positive.

We use the notation of Proposition 1. $\mathbb{V}^i$ is a convex set and must contain some strictly positive vectors as the one minimizing $g(\cdot)$, defined above, $\left(\mathbf{v}^i\right)^{\min} \equiv \Sigma^{-1}\boldsymbol{\omega}^i$, is strictly positive by assumption. Now we show that there it contains only a single point, the one for $v_{ii} = \frac{\omega_{ii}}{\Sigma_{ii}}$ and $v_{ij} = 0$ for all $i \neq j$, which is in any of the axes of R^n. This is sufficient to prove that vectors in $\mathbb{V}^i$ cannot have negative elements as for a convex set to cross any of the axes, it should have at least two points on that given axis. We show this by contradiction. Assume that $\mathbf{v}^i \in \mathbb{V}^i$ has an other elements on any of the axes, e.g. a $\bar{v}^i$ such that $v_{i1} = v_{i2} = ... = v_{in-1} = 0$ and $v_{in} = x$. Then $g(\cdot)$ simplifies to

$$g(\mathbf{v}^i) = x^2\Sigma_{nn} - 2x\omega_{in} + \frac{\omega^2_{ii}}{\Sigma_{ii}},$$

The function attains a minimum at $x^* = \frac{\omega_{in}}{\Sigma_{nn}}$. The minimum value of the function is $-\frac{\omega_{in}}{\Sigma_{nn}}$. Therefore $\mathbf{v}^i$ is not an element of $\mathbb{V}^i$ if

$$\frac{\omega^2_{in}}{\Sigma_{nn}} < \frac{\omega^2_{ii}}{\Sigma_{ii}}.$$

But this always holds in our parameterization (as $\frac{\omega^2_{in}}{\Sigma_{nn}} = \frac{\left(\rho\sigma^2_\theta\right)^2}{\sigma^2_\theta+\sigma^2_\varepsilon}$ and $\frac{\omega^2_{ii}}{\Sigma_{ii}} = \frac{\left(\sigma^2_\theta\right)^2}{\sigma^2_\theta+\sigma^2_\varepsilon}$).

As we showed in Proposition 1, for any network and any parameters V must be in the Cartesian product of $\mathbb{V}^{n\times n}$. However, the previous argument shows that there is a single matrix which has not strictly positive elements, the diagonal matrix with $v_{ii} = \frac{\omega_{ii}}{\Sigma_{ii}}$ for all i. But it is simple to check that this cannot be a fixed point of our system for any connected network and any parameters as long as $\rho \neq 0$.

Proof of Proposition 6

1. As $\rho \to 1$, we show that there exists an equilibrium such that

   $$\lim_{\rho\to 1} E\left(\theta^i | s^i, \mathbf{p}_{g^i}\right) = v^* \sum_{i=1}^n s^i, \ \forall i \in \{1, 2, ..., n\}$$

   where $v^* = \frac{\sigma^2_\theta}{n\sigma^2_\theta+\sigma^2_\varepsilon}$.

   If there exists an equilibrium in the OTC game, then it follows from the proof of Proposition 1 that

   $$E(\theta^i \left| s^i, \mathbf{p}_{g^i} \right.) = \bar{y}^i s^i + \sum_{k\in g^i} \bar{z}^i_{ik} E(\theta^k \left| s^k, \mathbf{p}_{g^k} \right.).$$

or

$$E(\theta^i \left| s^i, \mathbf{e}_{g^i} \right.) = \bar{y}^i s^i + \sum_{k \in g^i} \bar{z}^i_{ik} E(\theta^k \left| s^k, \mathbf{p}_{g^k} \right.).$$

Taking the limit as $\rho \to 1$, and using Case 2 in the proof of Proposition 1, we have that

$$\lim_{\rho \to 1} E\left(\theta^i | s^i, \mathbf{p}_{g^i}\right) = \frac{\sigma^2_\theta}{n\sigma^2_\theta + \sigma^2_\varepsilon} \sum_{i=1}^{n} s^i.$$

Given that

$$\lim_{\rho \to 1} E\left(\theta^i | s^i, \mathbf{p}_{g^i}\right)$$

The conditional variance is

$$\mathcal{V}\left(\theta^i | s^i, \mathbf{p}_{g^i}\right) = \sigma^2_\theta - \mathcal{V}\left(E\left(\theta^i | s^i, \mathbf{p}_{g^i}\right)\right)$$

and taking the limit $\rho \to 1$, we obtain

$$\lim_{\rho \to 1} \mathcal{V}\left(\theta^i | s^i, \mathbf{p}_{g^i}\right) = \sigma^2_\theta - \left(\frac{\sigma^2_\theta}{n\sigma^2_\theta + \sigma^2_\varepsilon}\right)^2 n\left(\sigma^2_\varepsilon + n\sigma^2_\theta\right).$$

and

$$\begin{aligned}
\lim_{\rho \to 1} \mathcal{V}\left(\theta^i | \mathbf{s}\right) &= \sigma^2_\theta - \mathcal{V}\left(E\left(\hat{\theta} | \mathbf{s}\right)\right) \\
&= \sigma^2_\theta - \left(\frac{\sigma^2_\theta}{n\sigma^2_\theta + \sigma^2_\varepsilon}\right)^2 n\left(\sigma^2_\varepsilon + n\sigma^2_\theta\right) \\
&= \sigma^2_\theta \frac{\sigma^2_\varepsilon}{n\sigma^2_\theta + \sigma^2_\varepsilon}
\end{aligned}$$

Proof of Proposition 7

Observe that $V = (I - Z)^{-1} Y$, for star network has the elements of

$$\begin{aligned}
v_{11} &= y_C \frac{1}{1 - (n-1) z_C z_P} \\
v_{i1} &= y_C \frac{z_P}{1 - (n-1) z_C z_P} \\
v_{ii} &= y_P \frac{1 - (n-2) z_C z_P}{1 - (n-1) z_C z_P} \\
v_{1i} &= y_P \frac{z_C}{1 - (n-1) z_C z_P} \\
v_{ij} &= y_P \frac{z_C z_P}{1 - (n-1) z_C z_P}
\end{aligned}$$

where y_C, y_P are the weights on the private signal and z_C, z_P are the weights on the others' guesses in the central and periphery agents' guessing function respectively. As maximizing

$E\left(-\left(\theta-e^{i}\right)^{2}\right)$ is equivalent with maximizing

$$2tr\left(V\Sigma_{\theta\mathbf{s}}\right)-tr\left(V\Sigma V^{\top}\right)$$

where $\Sigma_{ii}=1+\sigma^{2},\Sigma_{ij}=\rho,\left[\Sigma_{\theta s}\right]_{ii}=1,\left[\Sigma_{\theta s}\right]_{ij}=\rho$, we calculate the expressions for the components of this objective function.

$$\begin{aligned}\left[V\Sigma V^{\top}\right]_{11} &= \left(1+\sigma^{2}\right)v_{11}^{2}+\left(1+\sigma^{2}\right)(n-1)\,v_{1i}^{2}+\rho 2\,(n-1)\,v_{1i}v_{11}+\rho\,(n-1)\,(n-2)\,v_{1i}^{2}\\ &= \frac{\left(\left(1+\sigma^{2}\right)y_{C}^{2}+\left(\left(1+\sigma^{2}\right)+\rho(n-2)\right)(n-1)y_{P}^{2}z_{C}^{2}+\rho 2(n-1)y_{C}y_{P}z_{C}\right)}{\left(1-(n-1)z_{C}z_{P}\right)^{2}}\end{aligned}$$

and

$$\begin{aligned}\left[V\Sigma V^{\top}\right]_{ii} &= \left((n-2)\,(n-3)\,v_{ij}^{2}+(n-2)\,2\,(v_{i,1}+v_{i,i})\,v_{ij}+2v_{i,1}v_{i,i}\right)\rho+\left(\sigma^{2}+1\right)\left(v_{ii}^{2}+(n-2)\,v_{ij}^{2}+v_{i,1}^{2}\right)\\ &= \frac{\left(y_{C}+z_{C}y_{P}(n-2)\left(1-\frac{(n-1)}{2}z_{C}z_{P}\right)\right)2z_{P}y_{P}\rho+\left(\sigma^{2}+1\right)\left((1-(n-2)z_{C}z_{P})^{2}y_{P}^{2}+(n-2)y_{P}^{2}z_{P}^{2}z_{C}^{2}+y_{C}^{2}z_{P}^{2}\right)}{\left(1-(n-1)z_{C}z_{P}\right)^{2}}\end{aligned}$$

and

$$tr\left(V\Sigma V^{\top}\right)=\left[V\Sigma V^{\top}\right]_{11}+(n-1)\left[V\Sigma V^{\top}\right]_{ii}.$$

Also,

$$\begin{aligned}tr\left(V\Sigma_{\theta s}\right) &= v_{11}+(n-1)\,v_{ii}+\rho\,(n-1)\,(v_{1i}+v_{i1})+\rho\,(n-1)\,(n-2)\,v_{ij}=\\ &= \frac{y_{C}+\rho\,(n-1)\,y_{P}z_{C}}{\left(1-(n-1)\,z_{C}z_{P}\right)}+(n-1)\,\frac{y_{P}\left(1-(n-2)\,z_{C}z_{P}\,(1-\rho)\right)+\rho y_{C}z_{P}}{\left(1-(n-1)\,z_{C}z_{P}\right)}\end{aligned}$$

This implies that

$$\lim_{\delta\to 0}\frac{\partial U\left(z_{C}+\delta,z_{P}+\delta,y_{C}-\delta,y_{P}-\delta\right)}{\partial\delta}=-\frac{f\left(\bar{z}_{P},\bar{z}_{C},\bar{y}_{C},\bar{y}_{P};n,\rho,\sigma\right)}{\left(-1+(n-1)z_{C}z_{P}\right)^{3}},$$

where $f(\cdot)$ is a polynomial. Then we substitute in the analytical expressions for the decentralized optimum $z_{C}^{*},z_{P}^{*},y_{C}^{*},y_{P}^{*}$ given in closed form in Appendix B.2 and rewrite $\lim_{\delta\to 0}\frac{\partial U\left(z_{C}^{*}+\delta,z_{P}^{*}+\delta,y_{C}^{*}-\delta,y_{P}^{*}-\delta\right)}{\partial\delta}$ as a fraction. Both the numerator and the denominator are polynomials of σ^{2}.of order 9. A careful inspection reveals that each of the coefficients are positive for any $\rho\in(0,1)$ and $n\geq 3$.(Details on the resulting expressions in these calculations are available from the authors on request.)

Proof of Proposition 8

The first part comes by the observation that as $z_{V}\to 1-\frac{1}{n-1}$, $t_{V}\to\infty$, while t_{CN} is finite for these parameters. The second part comes from taking the limit $\rho\to 1$ of the ratio of the corresponding closed-form expressions we report in Appendices B.1 and B.3. In particular,

$$
\begin{aligned}
\lim_{\rho\to1}\frac{t_{CN}}{t_V} &= \frac{2n-3}{n-1} > 1\\
\lim_{\rho\to1}\frac{-\frac{\beta_{CN}}{2}E\left(p_{ij}^2\right)}{-\frac{\beta_V}{2}E\left(p_V^2\right)} &= \left(\frac{2n-3}{n-1}\right)^2 > 1\\
\lim_{\rho\to1}\frac{\frac{n(n-1)}{2}E\left(q_{CN}\left(e^i-p_{CN}\right)\right)}{E\left(q_V\left(e^i-p_V\right)\right)} &= \frac{2n-3}{(n-1)^2} < 1\\
\lim_{\rho\to1}\frac{\frac{n(n-1)}{2}E\left(q_{CN}\left(e^i-p_{CN}\right)\right)-\frac{\beta_{CN}}{2}E\left(p_{ij}^2\right)}{E\left(q_V\left(e^i-p_V\right)\right)-\frac{\beta_V}{2}E\left(p_V^2\right)} &= \frac{3-8n+4n^2}{(3(n-1)^2} > 1.
\end{aligned}
$$

Proof of Proposition 9

The statements come with simple algebra from the closed-form expressions we report in Appendix B.2.

Proof of Proposition 10

The first part comes by the observation that as $z_V \to 1-\frac{1}{n-1}$, $t_V\to\infty$, while t_C and t_P are finite for these parameters. The second part comes from taking the limit $\rho\to1$ of the ratio of the corresponding closed-form expressions we report in Appendices B.3 and B.2. In particular,

$$
\begin{aligned}
\lim_{\rho\to1}\frac{t_V}{t_C} &= (n-1)\frac{z_C+z_P-z_Cz_P}{(2-z_P)\left((n-1)z_V-(n-2)\right)} = \infty\\
\lim_{\rho\to1}\frac{t_V}{t_P} &= (n-1)\frac{z_C+z_P-z_Cz_P}{(2-z_C)\left((n-1)z_V-(n-2)\right)} = \frac{n-1}{n} < 1
\end{aligned}
$$

B Appendix: Closed forms in special cases

Throughout, we use the notation $\sigma \equiv \frac{\sigma_\varepsilon^2}{\sigma_\theta^2}$.

B.1 Centralized market

Following Vives (2011), we have

$$\begin{aligned} q_V &= t_V \left(e^i - p_V\right) \\ p_V &= \frac{t_V}{(\beta_V + n t_V)} \sum_i e^i, \end{aligned}$$

where $t_V = \frac{-\beta_V}{n(z_V - 1) + 2 - z_V}$ and $z_V = \frac{2\sigma\rho}{(1-\rho)(1+\rho(n-1))+\sigma}$ implying $\frac{\partial z_V}{\partial \rho}, \frac{\partial z_V}{\partial \sigma} > 0$ and expectations are privately fully revealing

$$e^i = E(\theta^i | s^i, \mathbf{e}_{g^i}) = E(\theta^i | \mathbf{s}) = \frac{1-\rho}{1+\sigma-\rho}\left(s^i + \frac{\rho\sigma}{(1-\rho)(1-\rho+n\rho+\sigma)} \sum_{i=1}^n s^i\right).$$

Substituting in these expressions into expressions (27)-(29) gives closed-form solutions for expected profit, expected utility of customers and welfare.

B.2 Star network

Without loss of generality, we characterize a star network with dealer 1 at the centre. There exist at least one equilibrium of the conditional guessing game such that for dealer 1

$$\bar{z}_{1i}^1 = \bar{z}_C \tag{B.1}$$

for any i. Similarly, for any dealer i in the periphery

$$\bar{z}_{i1}^i = \bar{z}_P.$$

We start with dealer 1, who chooses her demand function conditional on the beliefs of the other $(n-1)$ dealers. Given that she knows s_1, she can invert the signals of all the other dealers. Hence, her belief is given by

$$E(\theta^1 | s^1, \mathbf{e}_{g^1}) = E(\theta^1 | \mathbf{s}) = \frac{1-\rho}{1+\sigma^2-\rho}\left(s_1 + \frac{\rho\sigma^2}{(1-\rho)(1+\sigma^2-\rho+n\rho)} \sum_{i=1}^n s^i\right).$$

Or

$$E(\theta^1 | s^1, \mathbf{e}_{g^1}) = v_{11} s^1 + \sum_{j=2}^n v_{1j} s^j,$$

where

$$\begin{aligned}
v_{11} &= \frac{1-\rho}{1+\sigma^2-\rho}\left(1+\frac{\rho\sigma^2}{(1-\rho)\left(1+\sigma^2-\rho+n\rho\right)}\right) && \text{(B.2)}\\
v_{1j} &= \frac{1-\rho}{1+\sigma^2-\rho}\frac{\rho\sigma^2}{(1-\rho)\left(1+\sigma^2-\rho+n\rho\right)} && \text{(B.3)}
\end{aligned}$$

for all $j \neq 1$.

Further, the belief of a periphery dealer i is given by

$$E(\theta^i|s^i,e^1) = \begin{pmatrix} 1 \\ \tilde{\mathcal{V}}\left(\theta^i,e^1\right) \end{pmatrix}^\top \begin{pmatrix} 1+\sigma^2 & \tilde{\mathcal{V}}\left(s^i,e^1\right) \\ \tilde{\mathcal{V}}\left(s^i,e^1\right) & \mathcal{V}\left(e^1\right) \end{pmatrix}^{-1} \begin{pmatrix} s^i \\ e^1 \end{pmatrix},$$

where $\tilde{\mathcal{V}}(\cdot,\cdot) \equiv \frac{\mathcal{V}(\cdot,\cdot)}{\sigma_\theta^2}$ is the scaled covariance operator and

$$\tilde{\mathcal{V}}\left(e^1\right) = \frac{(1-\rho)\left(1+(n-1)\rho\right)+\sigma^2\left(1+(n-1)\rho^2\right)}{\left(1+\sigma^2-\rho\right)\left(1+\sigma^2+(n-1)\rho\right)}$$

$$\tilde{\mathcal{V}}\left(s^i,e^1\right) = \rho$$

$$\tilde{\mathcal{V}}\left(\theta^i,e^1\right) = \rho\frac{(1-\rho)\left(1+(n-1)\rho\right)+\sigma^2\left(2+(n-2)\rho\right)}{\left(1+\sigma^2-\rho\right)\left(1+\sigma^2+(n-1)\rho\right)}.$$

Since

$$\begin{aligned}
E(\theta^i|s^i,e^1) &= \frac{\tilde{\mathcal{V}}(e_1)-\tilde{\mathcal{V}}\left(\theta^i,e_1\right)\rho}{\tilde{\mathcal{V}}(e_1)\left(1+\sigma^2\right)-\rho^2}s^i + \frac{\tilde{\mathcal{V}}\left(\theta^i,e_1\right)\left(1+\sigma^2\right)-\rho}{\tilde{\mathcal{V}}(e_1)\left(1+\sigma^2\right)-\rho^2}e^1 \\
&= v_{ii}s^i + v_{i1}s^i + \sum_{\substack{j=2 \\ j\neq i}}^{n} v_{1j}s^j
\end{aligned}$$

for any $i \neq 1$, it follows that

$$\begin{aligned}
v_{i1} &= \frac{\tilde{\mathcal{V}}\left(\theta^i,e^1\right)\left(1+\sigma^2\right)-\rho}{\tilde{\mathcal{V}}\left(e^1\right)\left(1+\sigma^2\right)-\rho^2}v_{11} && \text{(B.4)}\\
v_{ii} &= \frac{\tilde{\mathcal{V}}\left(e^1\right)-\tilde{\mathcal{V}}\left(\theta^i,e^1\right)\rho}{\tilde{\mathcal{V}}\left(e^1\right)\left(1+\sigma^2\right)-\rho^2} + \frac{\tilde{\mathcal{V}}\left(\theta^i,e^1\right)\left(1+\sigma^2\right)-\rho}{\tilde{\mathcal{V}}\left(e^1\right)\left(1+\sigma^2\right)-\rho^2}v_{1j} && \text{(B.5)}\\
v_{ij} &= \frac{\tilde{\mathcal{V}}\left(\theta^i,e^1\right)\left(1+\sigma^2\right)-\rho}{\tilde{\mathcal{V}}\left(e^1\right)\left(1+\sigma^2\right)-\rho^2}v_{1j} && \text{(B.6)}
\end{aligned}$$

and

$$\begin{aligned}
\bar{y}_P &= \frac{\tilde{\mathcal{V}}\left(e^1\right)-\tilde{\mathcal{V}}\left(\theta^i,e^1\right)\rho}{\tilde{\mathcal{V}}\left(e^1\right)\left(1+\sigma^2\right)-\rho^2} \\
\bar{z}_P &= \frac{\tilde{\mathcal{V}}\left(\theta^i,e^1\right)\left(1+\sigma^2\right)-\rho}{\tilde{\mathcal{V}}\left(e^1\right)\left(1+\sigma^2\right)-\rho^2}.
\end{aligned}$$

Moreover, since

$$e^1 = E(\theta^1|s^1, \mathbf{e}_{g^1}) = \bar{y}_C s^1 + \sum_{j=2}^{n} \bar{z}_C e^j = \bar{y}_C s^1 + \sum_{j=2}^{n} \bar{z}_C \left(\bar{y}_P s^i + \bar{z}_P e^1\right),$$

then

$$E(\theta^1|s^1, \mathbf{e}_{g^1}) = \frac{\bar{y}_C}{1-(n-1)\,\bar{z}_C\bar{z}_P} s^1 + \sum_{j=2}^{n} \frac{\bar{z}_C\bar{y}_P}{1-(n-1)\,\bar{z}_C\bar{z}_P} s^i.$$

This implies that

$$\bar{z}_C = \frac{v_{1j}}{\bar{y}_P + (n-1)\,\bar{z}_P v_{1j}}$$

and

$$\bar{y}_C = \frac{v_{11}\bar{y}_P}{\bar{y}_P + (n-1)\,\bar{z}_P v_{1j}}.$$

We now solve the system (21) with substituting the expression for $\bar{z}_C, \bar{y}_C, \bar{z}_p, \bar{y}_P$ above giving the solution

$$z_P = 2\bar{z}_P$$

and

$$z_C = \bar{z}_C\left(n + 2\bar{z}_P - n\bar{z}_P - 1\right) + 1 - \sqrt{\left(\left(\bar{z}_C\left(n\left(1-\bar{z}_P\right) + 2\bar{z}_P - 1\right) + 1\right)\right)^2 - 4\bar{z}_C}$$

and

$$\begin{aligned} y_C &= \bar{y}_C\left(1 - n z_C \frac{2 - z_P}{4 - z_C z_P}\right) \\ y_P &= \bar{y}_P\left(1 - z_P \frac{2 - z_C}{4 - z_C z_P}\right). \end{aligned}$$

B.3 Complete network

In the complete network, each dealer i chooses her demand function conditional on the beliefs of the other $(n-1)$ dealers. Given that she knows s^i, she can invert the signals of all the other dealers. Hence, her belief is given by

$$E(\theta^i|s^i, \mathbf{e}_{g^i}) = E(\theta^i|\mathbf{s}) = \frac{1-\rho}{1+\sigma-\rho}\left(s^i + \frac{\rho\sigma}{(1-\rho)\,(1-\rho+n\rho+\sigma)} \sum_{i=1}^{n} s^i\right).$$

Then, following the same procedure as above (for a star), and taking into account that in a complete network trading strategies are symmetric, we obtain that

$$E(\theta^i|s^i, \mathbf{e}_{g^i}) = \bar{y}s^i + \bar{z}\sum_{\substack{j=1 \\ j\neq i}}^{n} e^j$$

where

$$e^j = E(\theta^j|s^j, \mathbf{e}_{g^j})$$

and

$$
\begin{aligned}
\bar{y} &= \frac{(1-\rho)(1+(n-1)\rho)}{1-\rho+\rho(1-\rho)(n-1)+\sigma(1+(n-2)\rho)} \\
\bar{z} &= \frac{\rho\sigma_\varepsilon^2}{\sigma_\theta^2+\sigma_\varepsilon^2+\rho^2\sigma_\theta^2-2\rho\sigma_\theta^2-2\rho\sigma_\varepsilon^2-n\rho^2\sigma_\theta^2+n\rho\sigma_\theta^2+n\rho\sigma_\varepsilon^2}.
\end{aligned}
$$

Solving the system (21), we obtain

$$
\begin{aligned}
y^i &= \frac{\sigma_\theta^2(1-\rho)(1+(n-1)\rho)}{\left(\sigma_\theta^2(1-\rho)(1+(n-1)\rho)+\sigma_\varepsilon^2(1+2(n-3)\rho)\right)+3\rho\sigma_\varepsilon^2}, \forall i \\
z_{ij}^i &= \frac{2\rho\sigma_\varepsilon^2}{\left(\sigma_\theta^2(1-\rho)(1+(n-1)\rho)+\sigma_\varepsilon^2(1+2(n-3)\rho)\right)+2\rho\sigma_\varepsilon^2}, \forall ij.
\end{aligned}
$$

Substituting in the expressions for t_{ij}^i in Proposition (2) we obtain

$$
t_{ij}^i = -\beta_{ij}\frac{\left(\sigma_\theta^2(1-\rho)(1+(n-1)\rho)+\sigma_\varepsilon^2(1+2(n-3)\rho)\right)+2\rho\sigma_\varepsilon^2}{2\rho\sigma_\varepsilon^2}.
$$